Homework 8 Solutions

Section 7.1

(10) $\int 3e^{1-x}\,dx$ with $u=1-x$

$u = 1-x$

$du = -dx \Rightarrow dx = -du$

$\int 3e^{1-x} = -\int 3e^{u}\,du$

$= -3e^{u} + C$

$= \boxed{-3e^{1-x} + C}$

(11) $\int xe^{-x^2/2}\,dx$ with $u = -x^2/2$

$u = -\frac{x^2}{2}$

$du = -x\,dx \Rightarrow x\,dx = -du$

$= \int e^{u}(-du) = -e^{u} + C$

$= \boxed{-e^{-x^2/2} + C}$

(12) $\int xe^{1-3x^2}dx$ with $u = 1-3x^2$

$u = 1-3x^2$

$du = -6x\,dx \Rightarrow x\,dx = -\frac{1}{6}du$

$= \int e^{u}(-\frac{1}{6}du)$

$= -\frac{1}{6}e^{u} + C$

$= \boxed{-\frac{1}{6}e^{1-3x^2} + C}$

(13) $\int \frac{x+2}{x^2+4x}\,dx$ with $u = x^2+4x$

$u = x^2+4x$

$du = (2x+4)\,dx$

$= 2(x+2)\,dx$

$\Rightarrow (x+2)\,dx = \frac{1}{2}du$

$= \int \frac{1}{u}(\frac{1}{2}du)$

$= \frac{1}{2}\ln|u| + C$

$= \boxed{\frac{1}{2}\ln|x^2+4x| + C}$

(14) $\int \frac{2x}{3-x^2}\,dx$ w/ $u = 3-x^2$

$u = 3-x^2$

$du = -2x\,dx$

$\Rightarrow 2x\,dx = -du$

$= \int \frac{1}{u}(-du) = -\ln|u| + C$

$= \boxed{-\ln|3-x^2| + C}$

(15) $\int \frac{3}{x+4}\,dx$ w/ $u = x+4$

$u = x+4$

$du = dx$

$= \int \frac{3}{u}du = 3\ln|u| + C$

$= \boxed{3\ln|x+4| + C}$

(16) $\int \frac{1}{5-x}\,dx$ with $u = 5-x$

$u = 5-x$

$du = -dx$

$\Rightarrow dx = -du$

so, $\int \frac{1}{5-x}\,dx = \int \frac{1}{u}(-du) = -\ln|u| + C$

$= \boxed{-\ln|5-x| + C}$

19. $\int (4x-3)\sqrt{2x^2-3x+2}\,dx$

Let $u = 2x^2-3x+2$

$du = (4x-3)\,dx$

$= \int u^{1/2}\,du$

$= \frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{2}{3}(2x^2-3x+2)^{3/2} + C}$

25. $\int 2xe^{x^2}\,dx$

Let $u = x^2$

Then $du = 2x\,dx$

$= \int e^u\,du$

$= e^u + C$

$= \boxed{e^{x^2} + C}$

26. $\int \cos x\, e^{\sin x}\,dx$

Let $u = \sin x$.

Then $du = \cos x\,dx$

$= \int e^u\,du = e^u + C$

$= \boxed{e^{\sin x} + C}$

29. $\int \sin\left(\frac{3\pi}{2}x + \frac{\pi}{4}\right)dx$

Let $u = \frac{3\pi}{2}x + \frac{\pi}{4}$

Then $du = \frac{3\pi}{2}\,dx$

$\Rightarrow dx = \frac{2}{3\pi}\,du$

$= \int \frac{2}{3\pi}\sin(u)\,du$

$= -\frac{2}{3\pi}\cos(u) + C$

$= \boxed{-\frac{2}{3\pi}\cos\left(\frac{3\pi}{2}x + \pi/4\right) + C}$

33. $\int \frac{(\ln x)^2}{x}\,dx$

let $u = \ln x$

Then $du = \frac{1}{x}\,dx$

$= \int u^2\,du$

$= \frac{1}{3}u^3 + C$

$= \boxed{\frac{1}{3}(\ln x)^3 + C}$

34. $\int \frac{dx}{(x-3)\ln(x-3)}$

let $u = \ln(x-3)$

Then $du = \frac{1}{x-3}\,dx$

$= \int \frac{1}{u}\,du = \ln|u| + C$

$= \boxed{\ln\left|\ln(x-3)\right| + C}$

37. $\int \frac{2ax+b}{ax^2+bx+c}\,dx$

let $u = ax^2+bx+c$

then $du = (2ax+b)\,dx$

$= \int \frac{du}{u} = \ln|u| + C$

$= \boxed{\ln|ax^2+bx+c| + C}$

38. $\int \frac{1}{ax+b}\,dx$

let $u = ax+b$

Then $du = a\,dx$

$\Rightarrow dx = \frac{1}{a}du$

$= \int \frac{1}{u}\left(\frac{1}{a}du\right)$

$= \frac{1}{a}\ln|u| + C$

$= \boxed{\frac{1}{a}\ln|ax+b| + C}$

48. $\int_{\ln 4}^{\ln 7} \frac{e^x}{(e^x-3)^2}\,dx$

let $u = e^x - 3$

Then $du = e^x\,dx$

if $x = \ln 4$, $u = e^{\ln 4} - 3$

$= 4-3 = 1$

if $x = \ln 7$, $u = e^{\ln 7} - 3$

$= 7-3 = 4$

$= \int_1^4 \frac{du}{u^2} = \int_1^4 u^{-2}\,du$

$= -u^{-1}\Big]_1^4$

$= -\frac{1}{u}\Big]_1^4$

$= -\frac{1}{4} - (-1)$

$= -1/4 + 1$

$= \boxed{3/4}$

50. $\int_{-\pi/6}^{\pi/6} \sin^2 x \cos x\,dx$

let $u = \sin x$

$du = \cos x\,dx$

if $x = -\pi/6$, $u = \sin(-\pi/6)$

$= -1/2$

if $x = \pi/6$, $u = \sin(\pi/6)$

$= 1/2$

$= \int_{-1/2}^{1/2} u^2\,du = \frac{1}{3}u^3\Big]_{-1/2}^{1/2}$

$= \frac{1}{3}(1/2)^3 - \frac{1}{3}\left(-\frac{1}{2}\right)^3$

$= \frac{1}{3}(1/8) + 1/3\,(1/8)$

$= \frac{2}{24} = \boxed{\frac{1}{12}}$

51. $\int_0^{\pi/4} \tan x \sec^2 x\,dx$

let $u = \tan x$

$du = \sec^2 x\,dx$

if $x = 0$, $u = \tan 0 = 0$

if $x = \pi/4$, $u = \tan \pi/4 = 1$

$= \int_0^1 u\,du = \frac{1}{2}u^2\Big]_0^1$

$= \frac{1}{2} - 0 = \boxed{\frac{1}{2}}$

52. $\int_0^{\pi/3} \frac{\sin x}{\cos^2 x}\,dx$

let $u = \cos x$

$du = -\sin x\,dx$

$\Rightarrow -du = \sin x\,dx$

if $x = 0$, $u = \cos 0 = 1$

if $x = \pi/3$, $u = \cos \pi/3 = 1/2$

$= \int_1^{1/2} \frac{-du}{u^2} = +\int_{1/2}^{1} u^{-2}\,du$

$= -u^{-1}\Big]_{1/2}^{1}$

$= -1 - \left(-\frac{1}{1/2}\right)$

$= -1 + 2 = \boxed{1}$

53. $\int_5^9 \frac{x}{x-3}\,dx$

let $u = x-3 \Leftrightarrow x = u+3$

$du = dx$

if $x=5$, $u = 5-3 = 2$

if $x = 9$, $u = 9-3 = 6$

$= \int_2^6 \frac{u+3}{u}\,du$

$= \int_2^6 \left(1 + \frac{3}{u}\right)du$

$= u + 3\ln|u|\Big]_2^6$

$= (6 + 3\ln(6)) - (2 + 3\ln(2))$

$= 4 + 3\ln(6) - 3\ln(2)$

$= 4 + 3\ln(6/2)$

$= \boxed{4 + 3\ln 3}$

54. $\int_0^2 \frac{x}{x+2}\,dx$

let $u = x+2 \Leftrightarrow x = u-2$

$du = dx$

if $x=0$, $u = 0+2 = 2$

if $x = 2$, $u = 2+2 = 4$

$= \int_2^4 \frac{u-2}{u}\,du$

$= \int_2^4 \left(1 - \frac{2}{u}\right)du$

$= u - 2\ln|u|\Big]_2^4$

$= 4 - 2\ln(4) - (2 - 2\ln(2))$

$= 2 - 2\ln(4) + 2\ln(2)$

$= 2 + 2\ln(2/4)$

$= 2 + 2\ln(1/2)$

$= 2 + 2(\ln(1) - \ln(2))$

$= \boxed{2 - 2\ln(2)}$

55. $\int_e^{e^2} \frac{dx}{x(\ln x)^2}$

let $u = \ln x$

then $du = \frac{1}{x}dx$

if $x = e$, then $u = \ln e = 1$

if $x = e^2$, then $u = \ln e^2 = 2$

$= \int_1^2 \frac{1}{u^2}\,du = -\frac{1}{u}\Big]_1^2$

$= -\frac{1}{2} - (-1)$

$= -\frac{1}{2} + 1$

$= \boxed{\frac{1}{2}}$

56. $\int_1^2 \frac{x\,dx}{(x^2+1)\ln(x^2+1)}$

let $u=\ln(x^2+1)$

then $du = \frac{2x}{x^2+1}dx$

$\Rightarrow \frac{1}{2}du = \frac{x\,dx}{x^2+1}$

if $x=1$, $u=\ln(2)$

if $x=2$, $u=\ln(5)$

$= \int_{\ln 2}^{\ln 5} \frac{1}{2}\cdot\frac{1}{u}\,du$

$= \frac{1}{2}\ln|u|\Big]_{\ln 2}^{\ln 5}$

$= \boxed{\frac{1}{2}\ln|\ln(5)| - \frac{1}{2}\ln|\ln(2)|}$

57. $\int_1^9 \frac{1}{\sqrt{x}}e^{-\sqrt{x}}\,dx$

let $u=\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx \Rightarrow 2du = \frac{1}{\sqrt{x}}dx$

if $x=1$, $u=1$

if $x=9$, $u=\sqrt{9}=3$

$= \int_1^3 2e^{-u}\,du$

$= -2e^{-u}\Big]_1^3$

$= -2e^{-3} - (-2e^{-1})$

$= \boxed{-2e^{-3}+2e^{-1}}$

58. $\int_0^2 x\sqrt{4-x^2}\,dx$

let $u=4-x^2$

then $du=-2x\,dx$

$\Rightarrow -\frac{1}{2}du = x\,dx$

if $x=0$, $u=4-0=4$

if $x=2$, $u=4-2^2=0$

$= \int_4^0 -\frac{1}{2}u^{1/2}\,du$

$= \int_0^4 \frac{1}{2}u^{1/2}\,du$

$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big]_0^4$

$= \frac{1}{3}u^{3/2}\Big]_0^4$

$= \frac{1}{3}(4)^{3/2} - 0 = \frac{1}{3}(8) = \boxed{\frac{8}{3}}$