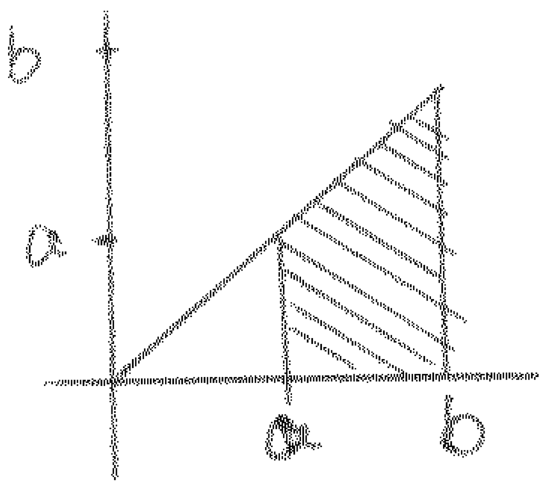
Calculus I Assignment 7

6.1

(39)



area of big triangle $= \frac{b^2}{2}$

area of small triangle $= \frac{a^2}{2}$

area under $y=x$ from a to b = area of big triangle − area of small triangle

$= \frac{b^2-a^2}{2}$

(61) $\int_{-2}^{3} |x|\,dx =$ area of triangle $(-2,0)(0,0),(-2,2)$ + area of triangle $(0,0),(3,0),(3,3)$

$= \frac{2^2}{2} + \frac{3^2}{2} = \frac{13}{2} = 6.5$

(62) half area of disc centered at $(0,0)$, with radius $3 = \frac{9}{2}\pi$

(68) (a) $\int_0^2 \frac{1}{2}x^2\,dx = \frac{1}{2}\int_0^2 x^2\,dx = \frac{1}{2}\cdot\frac{1}{3}2^3 = \frac{4}{3}$

(b) $\int_{-3}^{0} 2x^2\,dx = -\int_0^{-3} 2x^2\,dx = -2\cdot\int_0^{-3} x^2\,dx = -2\cdot\frac{1}{3}(-3)^3 = 18$

(c) $\int_1^3 \frac{1}{3}x^2\,dx = \int_0^3 \frac{1}{3}x^2\,dx - \int_0^1 \frac{1}{3}x^2\,dx = \frac{1}{3}\int_0^3 x^2\,dx - \frac{1}{3}\int_0^1 x^2\,dx = \frac{1}{3}\cdot\frac{1}{3}3^3 - \frac{1}{3}\cdot\frac{1}{3}1^3 = \frac{26}{9}$

(70) $\int_{-3}^{-3} e^{-x^2/2}\,dx = 0$ for $a=b$

6.2

(2) $\frac{d}{dx}\int_0^x (1-u^3)\,du = 1-x^3$. by FTC, since function $1-u^3$ is continuous between 0 and x

(4) $\frac{d}{dx}\int_0^x (1+u^4)\,du = 1+x^4$

(6) $\frac{d}{dx}\int_0^x \sqrt{1+u^2}\,du = \sqrt{1+x^2}$ $(x>0)$

(10) $\frac{d}{dx}\int_1^x u e^{-u^2}\,du = xe^{-x^2}$

(40) $\int (x^3-4)\,dx = \frac{1}{4}x^4 - 4x + C$

(50) $\int (x^{3/5}+x^{5/3})\,dx = \frac{x^{8/5}}{8/5} + \frac{x^{8/3}}{8/3} + C = \frac{5}{8}x^{8/5} + \frac{3}{8}x^{8/3} + C$

(60) $\int 2e^{-x/3}\,dx = \frac{2}{-1/3}e^{-x/3} + C = -6e^{-x/3}$

(98) $\int_{-1}^{3} (2x^2-1)\,dx = \left[\frac{2}{3}x^3 - x\right]_{-1}^{3} = \left(\frac{2}{3}3^3 - (3)\right) - \left(\frac{2}{3}(-1)^3 - (-1)\right) = (18-3) - \left(-\frac{2}{3}+1\right)$

$= 15 - \frac{1}{3} = \frac{44}{3}$

(100) $\int_1^2 x^{5/2}\,dx = \frac{2}{7}x^{\frac{7}{2}}\Big|_1^2 = \frac{2}{7}\left(2^{\frac{7}{2}} - 1^{\frac{7}{2}}\right) = \frac{2(\sqrt{2^7}-1)}{7} = \frac{2(8\sqrt{2}-1)}{7} = \frac{16\sqrt{2}-2}{7}$

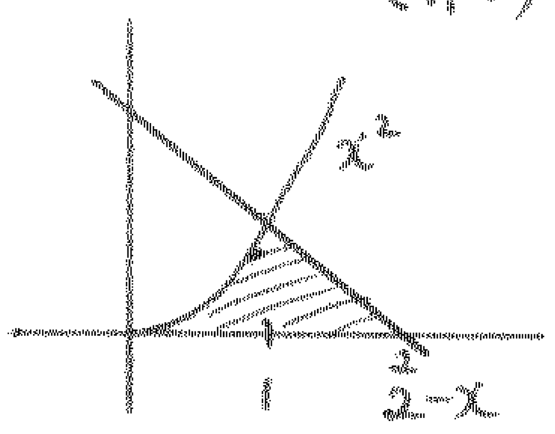
(115) $\int_{-1}^{0} e^{3x}\,dx = \frac{1}{3}e^{3x}\Big|_{-1}^{0} = \frac{1}{3}e^0 - \frac{1}{3}e^{-3} = \frac{1-e^{-3}}{3}$

(124) $\lim_{h\to 0} \frac{1}{h}\int_0^h e^x\,dx = \lim_{h\to 0} \frac{\int_0^h e^x\,dx}{h} \underset{\left(\frac{0}{0}\right)}{\overset{L'H}{=}} \lim_{h\to 0} \frac{e^h}{1} = e^0 = 1$

$\boxed{6.3}$

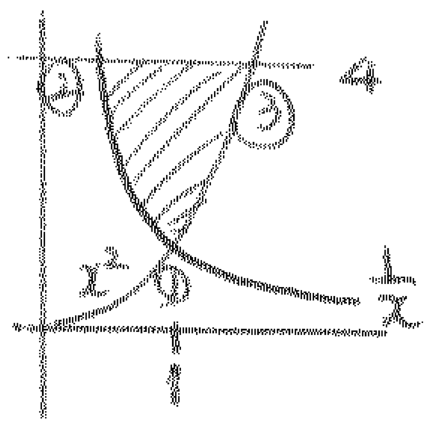
⑥ $y=x^2,\ y=2-x,\ y=0$ (first quadrant)

intersection: $(1,1)$ $\quad (x^2=2-x,\quad x^2+x-2=0 \Rightarrow (x+2)(x-1)=0 \Rightarrow x=-2 \text{ or } x=1,\ x>0)$



$$\text{Area} = \int_0^1 x^2\,dx + \int_1^2 (2-x)\,dx$$
$$= \left[\frac{x^3}{3}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2 = \frac{1}{3} + \left(2\cdot 2 - \frac{4}{2}\right) - \left(2 - \frac{1}{2}\right)$$
$$= \frac{1}{3} + 4 - 2 - 2 + \frac{1}{2} = \frac{5}{6}$$

⑦ $y=x^2,\ y=\frac{1}{x},\ y=4$ (first quadrant)



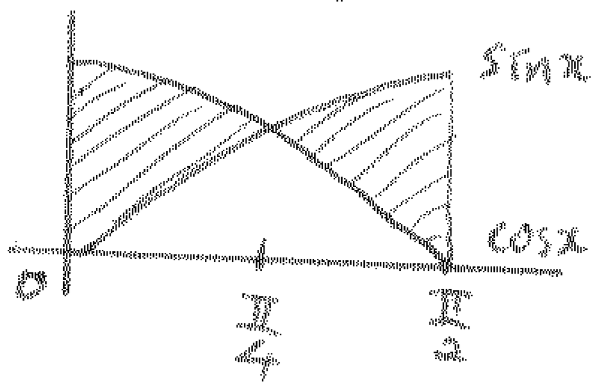
intersection: ① $x^2 = \frac{1}{x} \Rightarrow x^3 = 1 \Rightarrow x=1 \quad \therefore (1,1)$

② $\frac{1}{x} = 4 \Rightarrow x = \frac{1}{4} \quad : (\frac{1}{4}, 4)$

③ $x^2 = 4 \Rightarrow x = 2. \quad : (2,4)$

$$\text{area} = \int_{\frac{1}{4}}^1 \left(4 - \frac{1}{x}\right)dx + \int_1^2 (4-x^2)\,dx = \Big[4x - \ln x\Big]_{\frac{1}{4}}^1 + \left[4x - \frac{x^3}{3}\right]_1^2$$
$$= (4-0) - \left(1 - \ln\frac{1}{4}\right) + \left(8 - \frac{8}{3}\right) - \left(4 - \frac{1}{3}\right)$$
$$= 7 - \frac{8}{3} + \frac{1}{3} + \ln\frac{1}{4} = 7 - \frac{7}{3} - \ln 4 = \frac{14}{3} - \ln 4$$

⑧ $y=\sin x,\quad y=\cos x$ $\quad$ from $x=0$ to $x=\frac{\pi}{2}$



intersection: $\frac{\pi}{4}$

$$\text{Area} = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)\,dx$$
$$= \Big[\sin x + \cos x\Big]_0^{\frac{\pi}{4}} + \Big[-\cos x - \sin x\Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$
$$= \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - (\sin 0 + \cos 0) + \left(-\cos\frac{\pi}{2} - \sin\frac{\pi}{2}\right) - \left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)$$
$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 0 - 1 - 0 - 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$
$$= 2\sqrt{2} - 2$$

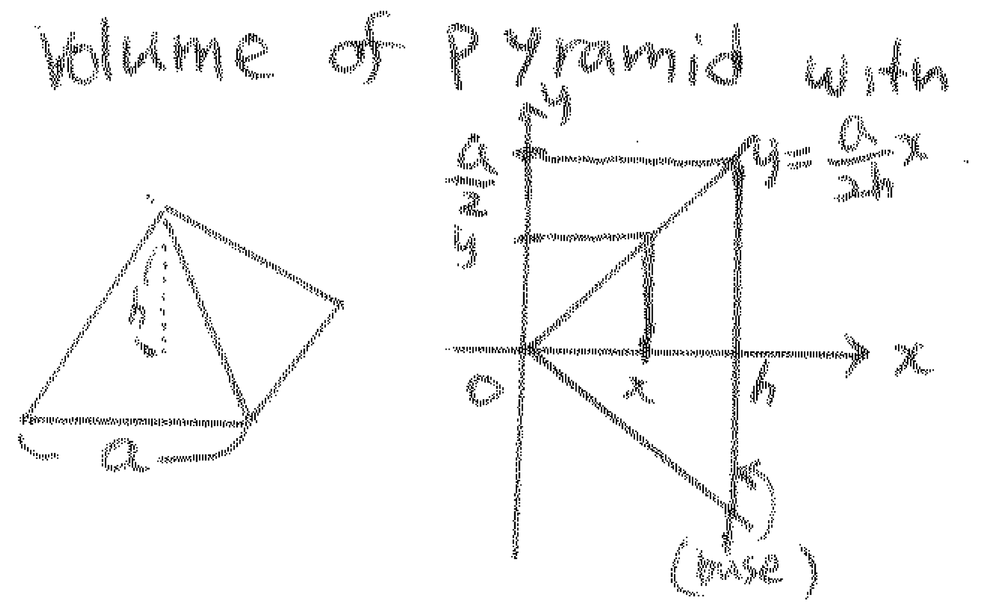
㉑ $\frac{dl}{dt}$ = growth rate of an organism at time t (month)

$\int_2^7 \frac{dl}{dt}\,dt$: cumulative growth of organism between months $[2, 7]$

㉒ $\frac{dw}{dx}$: rate of change of the weight of an organism of age x.

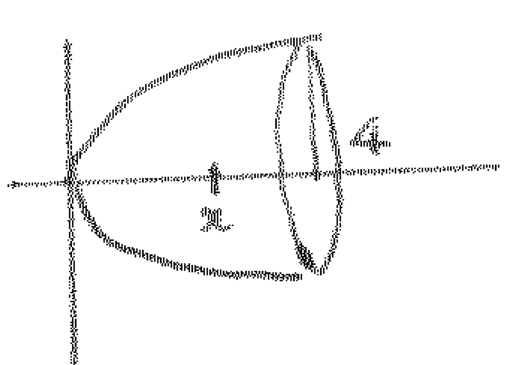
$\int_3^5 \frac{dw}{dx}\,dx$: cumulative change of weight between age 3 and age 5.

㉞ volume of Pyramid with square base of sidelength a and height h.



$$\text{Volume} = \int_0^h (\text{Area of square at } x)\,dx$$
$$= \int_0^h (2y)^2\,dx = \int_0^h \left(\frac{a}{h}x\right)^2 dx = \frac{a^2}{h^2}\int_0^h x^2\,dx$$
$$= \frac{a^2}{h^2}\left[\frac{x^3}{3}\right]_0^h = \frac{a^2h^3}{3h^2} = \frac{a^2h}{3}$$

㊱ $y=\sqrt{x},\ y=0,\ x=4.$ $\quad$ rotation about x-axis.



$$\text{Volume} = \int_0^4 (\text{Area at } x)\,dx$$
$$= \int_0^4 \pi y^2\,dx = \pi\int_0^4 x\,dx = \pi\,\frac{x^2}{2}\Big|_0^4 = \pi\cdot\frac{16}{2} = 8\pi$$