# Calc 106 - Homework 4 Solutions

## Section 4.4

63. Find $\frac{dy}{dt}$ when:

$x^2 y = 1$

$\frac{dx}{dt} = 3$ for $x = 2$

If $x = 2$, $4y = 1$
$y = 1/4$

$$x^2 y = 1$$
$$2x\frac{dx}{dt}y + x^2\frac{dy}{dt} = 0$$
$$2(2)(3)(1/4) + (4)\frac{dy}{dt} = 0$$
$$3 + 4\frac{dy}{dt} = 0$$
$$\boxed{\frac{dy}{dt} = \frac{-3}{4}}$$

64. Find $\frac{du}{dt}$ when:

$u^2 + v^3 = 12$

$\frac{dv}{dt} = 2$ for $v = 2$

$u > 0$

If $v = 2$, $u^2 + 8 = 12$
$u^2 = 4$
$u = 2$

$$u^2 + v^3 = 12$$
$$2u\frac{du}{dt} + 3v^2\frac{dv}{dt} = 0$$
$$2(2)\frac{du}{dt} + 3(4)(2) = 0$$
$$4\frac{du}{dt} + 24 = 0$$
$$\boxed{\frac{du}{dt} = -6}$$

65. $V = x^3$

$$\boxed{\frac{dV}{dt} = 3x^2\frac{dx}{dt}}$$

69. $V = \pi r^2 h = 25\pi h$

$r = 5m$

$\frac{dV}{dt} = 250 \text{ liters/min} \cdot \frac{1 m^3}{1{,}000 \text{ liters}}$
$= \frac{1}{4} m^3/\text{min}.$

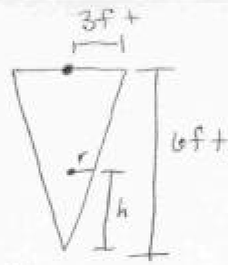
$\frac{dh}{dt} = ?$

$$\frac{dV}{dt} = 25\pi\frac{dh}{dt}$$
$$\frac{1}{4} = 25\pi\frac{dh}{dt}$$
$$\frac{1}{100\pi} = \frac{dh}{dt}$$
$$\boxed{\frac{dh}{dt} = \frac{1}{100\pi} \text{ m/min.}}$$

2.

70.

$V = \frac{1}{3}\pi r^2 h$




$\frac{r}{h} = \frac{3}{6} = \frac{1}{2}$

$r = \frac{1}{2}h$

$\frac{dV}{dt} = 5 ft^3/min$

$\frac{dh}{dt} = ?$

so $V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{12}\pi \cdot 3h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

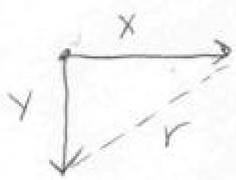
$5 = \frac{1}{4}\pi(2)^2 \frac{dh}{dt}$

$5 = \frac{1}{4}\pi(4)\frac{dh}{dt}$

$5 = \pi \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{5}{\pi} \text{ ft/min.}}$

71.




$\frac{dx}{dt} = 15$ mph

$\frac{dy}{dt} = 18$ mph

$\frac{dr}{dt} = ?$

After 20 minutes, $x=5$, $y=6$

$\therefore r = \sqrt{5^2+6^2}$

$= \sqrt{25+36}$

$= \sqrt{61}$

$x^2+y^2=r^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$

$\cancel{2}(5)(15) + \cancel{2}(6)(18) = \cancel{2}(\sqrt{61})\frac{dr}{dt}$

$75 + 108 = \sqrt{61}\frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{183}{\sqrt{61}} \text{ mph}}$

After 40 minutes, $x=10$, $y=12$

$r = \sqrt{10^2+12^2}$

$= \sqrt{100+144}$

$= \sqrt{244}$

$= 2\sqrt{61}$

$x^2+y^2=r^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$

$\cancel{2}(10)(15) + \cancel{2}(12)(18) = \cancel{2}(2\sqrt{61})\frac{dr}{dt}$

$150 + 216 = 2\sqrt{61}\frac{dr}{dt}$

$366 = 2\sqrt{61}\frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{183}{\sqrt{61}} \text{ mph}}$

## Section 4.5

60. $y = \cos^2 x$

At what points does $y = \cos^2 x$ have a horizontal tangent?

where the derivative is 0.

$y = \cos^2 x$

$\frac{dy}{dx} = 2\cos x(-\sin x)$

Set $-2\cos x \sin x = 0$

$\cos x \sin x = 0$

$\cos x = 0$ or $\sin x = 0$

$x = \frac{\pi}{2} + \pi k$ $\quad x = \pi k$

k an integer $\quad$ k an integer

63. $\frac{d}{dx}(\sec x) = \frac{d}{dx}[(\cos x)^{-1}]$

$= -1\cdot(\cos x)^{-2}\cdot(-\sin x)$

$= \frac{\sin x}{\cos^2 x}$

$= \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}$

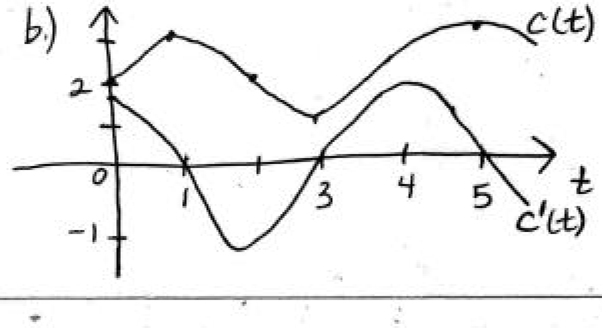
$= \tan x \sec x$

73. $c(t) = 2 + \sin(\frac{\pi}{2}t)$

a) $\frac{dc}{dt} = 0 + \cos(\frac{\pi}{2}t)\cdot\frac{\pi}{2}$

$= \frac{\pi}{2}\cos(\frac{\pi}{2}t)$

b.)




c.) i) $\frac{dc}{dt} = 0$

ii) $c(t)$ is increasing

iii) $c(t)$ is a local maximum or minimum

($c(t)$ has a horizontal tangent)

# Section 4.6

59. $N(t) = e^{2t}, \; t \geq 0$

a.) $N(0) = e^{2(0)} = 1$

b.) $\frac{dN}{dt} = 2 \cdot \underbrace{e^{2t}}_{N(t)}$

$\frac{dN}{dt} = 2N$

60. $N(t) = N_0 e^{rt}, \; t \geq 0$

a.) $N(0) = N_0 e^{r(0)}$
$= N_0 (1)$
$= N_0$

b.) $\frac{dN}{dt} = N_0 \cdot r e^{rt}$
$= r \underbrace{N_0 e^{rt}}_{N(t)}$
$= rN$

61. $N(t) = N(0) 2^t$

$\frac{dN}{dt} = N(0) \cdot \ln 2 \cdot 2^t$
$= \ln 2 \cdot \underbrace{N(0) 2^t}_{N(t)}$
$= \ln 2 \cdot N$

so, $\boxed{\frac{dN}{dt} = \ln 2 \cdot N}$

Therefore, $\frac{dN}{dt}$ is proportional to N.

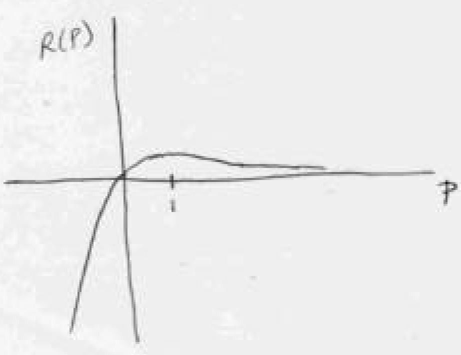
62. $N(t) = N(0) 2^t$

From 61.) $\frac{dN}{dt} = \ln 2 \cdot N$

Per capita growth rate is $\frac{1}{N} \frac{dN}{dt} = \ln 2$

64. $R(P) = \alpha P e^{-\beta P}, \; P \geq 0$

a.) Sketch R(P) when $\beta = 1$ and $\alpha = 2$.




b.) $R(P) = 2Pe^{-P}$

$R'(P) = 2[1 \cdot e^{-P} + P \cdot (-e^{-P})]$
$= 2[e^{-P} - Pe^{-P}]$

$R(P) = \alpha P e^{-\beta P}$

$R'(P) = \alpha[e^{-\beta P} + P(-\beta e^{-\beta P})]$
$= \alpha[e^{-\beta P} - \beta P e^{-\beta P}]$

c.) $R'(P) = 2[e^{-P} - Pe^{-P}] = 0$
$e^{-P} = Pe^{-P}$
$P = 1$

$\alpha[e^{-\beta P} - \beta P e^{-\beta P}] = 0$
$e^{-\beta P} = \beta P e^{-\beta P}$
$\frac{1}{\beta} e^{-\beta P} = P e^{-\beta P}$
$P = 1/\beta$

5.

## Problem from Handout

$$2x^3 + 2y^3 = 9xy$$

$$2(2^3) + 2(1^3) = 9(2)(1)$$
$$2(8) + 2 = 18$$
$18 = 18$, so $(2,1)$ is on the curve

$$6x^2 + 6y^2\frac{dy}{dx} = 9\left[1\cdot y + x\frac{dy}{dx}\right]$$

$(x,y)=(2,1)$

$$6x^2 + 6y^2\frac{dy}{dx} = 9y + 9x\frac{dy}{dx}$$
$$6y^2\frac{dy}{dx} - 9x\frac{dy}{dx} = 9y - 6x^2$$
$$\frac{dy}{dx} = \frac{9y-6x^2}{6y^2-9x}$$
$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{9(1)-6(2^2)}{6(1^2)-9(2)} = \frac{9-24}{6-18} = \frac{-15}{-12} = \boxed{5/4}$$

$\frac{dy}{dx}$ is undefined when $6y^2 - 9x = 0$

$$\Rightarrow 6y^2 = 9x$$
$$y^2 = \tfrac{3}{2}x$$
$$y = \sqrt{\tfrac{3}{2}x} \text{ or } x = \tfrac{2}{3}y^2$$

$6y^2 - 9x$ intersects the curve where?

$$2\left(\tfrac{2}{3}y^2\right)^3 + 2y^3 = {}^{3}\cancel{9}\left(\tfrac{2}{\cancel{3}}y^2\right)y$$
$$2\left(\tfrac{8}{27}y^6\right) + 2y^3 = 6y^3$$
$$\tfrac{16}{27}y^6 + 2y^3 = 6y^3$$
$$\tfrac{16}{27}y^6 - 4y^3 = 0$$
$$y^3 = \frac{4 \pm \sqrt{16 - 4(16/27)(0)}}{2(16/27)}$$
$$= \frac{4 \pm 4}{32/27}$$

$y^3 = 0 \Rightarrow y = 0$ or $y^3 = \frac{27}{32}(8) = \frac{27}{4} \Rightarrow y = \sqrt[3]{\frac{27}{4}} = \frac{3}{\sqrt[3]{4}} \Rightarrow x = \frac{2}{3}\left(\frac{3}{\sqrt[3]{4}}\right)^2 = \frac{2}{3}\left(\frac{9}{\sqrt[3]{16}}\right) = \frac{6}{\sqrt[3]{16}}$

6.

So, $6y^2 - 9x$ intersects the curve at $(0,0)$
and $\left(\frac{6}{\sqrt[3]{16}}, \frac{3}{\sqrt[3]{4}}\right) \approx (2.38, 1.89)$

At $(0,0)$, cannot draw a tangent line
At the second point, there is a vertical tangent.