

Assignment 1--The derivative as scientific concept, and some limit problems.

Please read sections 3.1, 3.2, and 3.3.

Please do problems 3.1. {5,10,17,33,34,36,37,38}

For each problem below, answer in plain English. I suggest you write your answers on a hardcopy of this worksheet.

Problem 1: Let p be the position (in meters) of a particle, and let t be time (**in seconds**). What does it mean to say that the derivative of p with respect to t equals 4? This means that the rate of change of the position of the particle with respect to time is 4 meters per second, i.e. the velocity is 4 m/s. In other words, any change in time causes a change 4 times as large in the position. Since this rate of change is constant for all t , it is also ok to say that for every additional second in time, the particle advances 4 meters.

Problem 2: Let I be the daily intake (in grams) of a certain drug, and let S be the resulting patient survivability (in expected years of life remaining). What does it mean to say that the derivative of S with respect to I equals -0.5? Do you recommend this drug? The rate of change of survivability with respect to the daily intake of the drug is -0.5. In other words, any **increase** in drug amount (in grams) causes life expectancy to **decrease** (in years) by .5 times that increase. Again, since this is a constant rate of change, it is ok to say that an increase of 1 gram causes a decrease of half a year of life expectancy.

Problem 3: Let x be the number of stitches ~~required to repair~~ **performed on** a just-damaged garment, and let y be the number of stitches required to repair the same garment if it is long neglected (**instead of receiving x stitches**). What does it mean to say that the derivative of y with respect to x is -9.0? For every stitch you perform right away, you save yourself nine stitches that you would have to perform later if you neglect the garment, i.e. "a stitch in time saves nine!"

Problem 4: One evening, you decide to drive from Warm Valley to Cold Peak. Let t be time in hours, x be position in miles, and y be the temperature of the air.

1. Translate: The derivative of x with respect to t is 60 - the rate of change of position w.r.t. time is 60 miles/hour, i.e. the velocity is 60 mph.
2. Translate: The derivative of y with respect to x is -0.1 - the rate of change of temperature with respect to position is -0.1 degrees per hour, so any increase in position causes a temp. drop of -0.1 times that amount in degrees (move 1 mile, temp. drops 0.1 degree.)
3. What therefore is the value of dy/dt , and what does that value mean? $dy/dt = -6$ This means that an increase in time causes a drop in temperature that is 6 times larger than the change in time. So the temp. drops -6 degrees every hour.

4. State explicitly how you determined dy/dt from the other two derivatives. Since we know that a change in t causes x to change by a factor of 60, and a change in x causes y to change by a factor of -0.1, if we multiply the two values, we will get the change in y with respect to t : $(dx/dt)(dy/dx)=(60 \text{ miles/hour}) \times (-0.1 \text{ degrees/mile}) = -6 \text{ degrees/hour}$. Notice how the units cancel as needed.

Problem 5: Shivering is a way of converting stored biochemical energy to heat to protect against cold weather. Temperature T , of course, is naturally measured in degrees. Severity of shivering could be measured by the number (M) of muscle groups recruited to shiver.

For a cold person, should dM/dT be positive, negative, or zero? Explain. It should be negative since any increase in temp. would cause the number of muscles shivering to decrease, therefore the graph would have negative slope.

What about for a very warm person? A very warm person doesn't shiver at all, so there is no change in the number of muscles shivering as long as the temp. change is very small. dM/dT is zero - note this is because there is no change in the # of muscles shivering, not because the number of muscles shivering is zero.

Problem 6: Let $E=f(x)$ be the elevation in feet of the Mississippi river x miles from its source. What are the units of dE/dx ? What can you say about the sign of dE/dx ? Using figures (available online) for St. Louis, MO and Memphis, TN, estimate dE/dx . Using this data: elev. of St. Louis - 400 ft. , elev. of Memphis - 250, distance from St. Louis to Memphis - 200 miles

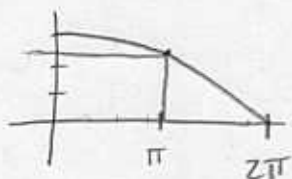
The units for dE/dx are feet/mile. The sign should be negative since as your distance from the source increases, the elevation decreases (i.e. water flows downhill!). A gross estimate for $dE/dx = (250-400)/200 = -0.75$. Notice that I subtracted the elev. of St. Louis from the elev. of Memphis - you always want to think of these problems in terms of increasing the x variable, in this case distance from the source.

Problem 7: The time (in minutes) $T=f(C)$ required for a chemical reaction depends on the quantity (in grams) C of catalyst present.

1. Translate: $f(5)=9$
When 5 grams of catalyst are present, the chemical reaction takes 9 minutes.
2. Translate: $f'(5)=-1$ When you have 5 grams of catalyst, the instantaneous rate of change of the chemical reaction time with respect to amount of catalyst is -1. So, if you change the amount of catalyst very slightly from 5 grams, say to 5.001 grams, the time required will decrease by approx. $(-1)(.001)=-.001$. This doesn't tell you anything about the rate of change at other values of C , so this does **not** mean that for every increase in grams, you get a decrease of one minute.

$$3.1 = 5, 10, 17, 33, 34, 36, 37, 38$$

$$5. \lim_{x \rightarrow \pi} 3 \cos \frac{x}{4}$$



The graph of $f(x)$ shows that
 $\lim_{x \rightarrow \pi} 3 \cos \frac{x}{4} = \frac{3\sqrt{2}}{2} \approx 2.12$

$$10. \lim_{x \rightarrow 0} \frac{e^x + 1}{2x + 3} = .66 = 2/3$$

Table

x	.1	.01	.001	-.001	-.01	-.1
f(x)	.6579	.6656	.6666	.6668	.6678	.6803

$$17. \lim_{x \rightarrow \infty} \frac{x}{1+x} = 1$$



The graph shows a horizontal asymptote at $y=1$ as $x \rightarrow \infty$.

$$33. f(x) = \frac{2}{x^2}$$

Table

x	-10^6	-10^5	-10^3	-1	.1	.001	0	.001	.1	1	10^3	10^5	10^6
f(x)	2×10^{-12}	2×10^{-10}	2×10^{-6}	2	200	$2,000,000$	-	$2,000,000$	200	2	2×10^6	2×10^{10}	2×10^{12}

$$\text{as } x \rightarrow \infty, f(x) \rightarrow 0$$

$$\text{as } x \rightarrow -\infty, f(x) \rightarrow 0$$

$$\text{as } x \rightarrow 0, f(x) \rightarrow +\infty$$

$$34. f(x) = \frac{2x}{x-1}$$

x	10^5	10^3	10^2	.9	.99	.999	1	1.001	1.01	1.1	10^2	10^3	10^5
f(x)	1.999	1.998	1.980	1.8	1.98	1.99	-	2.002	2.02	2.2	2.02	2.002	2.00002

$$x \rightarrow \infty \Rightarrow f(x) \rightarrow 2$$

$$x \rightarrow -\infty \Rightarrow f(x) \rightarrow 2$$

$$x \rightarrow 1^+ \Rightarrow f(x) \rightarrow \infty$$

$$x \rightarrow 1^- \Rightarrow f(x) \rightarrow -\infty$$

$\lim_{x \rightarrow 1} f(x)$ d.n.e.

$$36. \lim_{x \rightarrow 0} \cos \frac{1}{x}$$

diverges by
oscillations



$$\begin{aligned} 37. \lim_{x \rightarrow -1} (x^3 + 7x - 1) &= \left(\lim_{x \rightarrow -1} x \right)^3 + 7 \left(\lim_{x \rightarrow -1} x \right) - 1 \\ &= (-1)^3 + 7(-1) - 1 \\ &= -1 - 7 - 1 \\ &= -9 \end{aligned}$$

$$\begin{aligned} 38. \lim_{x \rightarrow 2} (3x^4 - 2x + 1) &= 3 \left(\lim_{x \rightarrow 2} x \right)^4 - 2 \left(\lim_{x \rightarrow 2} x \right) + 1 \\ &= 3(2)^4 - 2(2) + 1 \\ &= 48 - 4 + 1 \\ &= 45 \end{aligned}$$