MATH 106 — SECOND EXAM

DEPARTMENT OF MATHEMATICS
Johns Hopkins University

April 6, 2004

NAME: _____ SIGNATURE: _____

TA (circle): Ann Stewart, Grace Currie

1. This exam has several pages including this cover. There are several questions.
2. Use of books, notes, or scratch paper is not allowed. You may certainly use a calculator (but not its manual).
3. **Show all of your work!** Partial credit is available for many problems but can only be given if the graders understand your work. Be sure to explain your reasoning carefully. Include units in your answers whenever appropriate.
4. Read directions carefully. For some problems, a brief answer is sufficient, but others require you to show all work or give explanations.
5. For exam security reasons, students of Dr. Budur *may not leave before 11:05.* We apologize for any inconvenience this may cause.

PROBLEM	POINTS	SCORE
1	20	
TOTAL	100	

1. (20 points) The function $q(x)$ is graphed below. For each of the items listed, calculate it exactly. If you cannot calculate it exactly, estimate as well as possible.

a. (4 points) $q(2) =$

b. (4 points) $q'(2) =$

c. (4 points) $\int_1^4 q(x)dx$

d. (4 points) $\lim_{n\to\infty} \Sigma_{i=0}^{n-1} q(3+\frac{i}{n})\frac{1}{n} =$

e. (4 points) Assume that $Q'(x) = q(x)$. Then $Q(4) - Q(1) =$

2. (12 points) Calculate the limits. To get full credit, you must use a reliable, exact method. Calculator shortcuts don't count!

a. $\lim_{x \to 0} \dfrac{3x^2 + 4x}{e^{2x}}$

b. $\lim_{x \to \infty} \sqrt{x^2 - 8x + 19} - x$

c. $\lim_{x \to 0} x^x$

3. A crystal appears inside a saltwater solution and grows at a rate of $R(t) = 4 + \sqrt{(}x) + \frac{x^3}{16}$ grams per minute, where t is the time in minutes since the crystal was first noticed by the experimenter. When the crystal was first noticed, it already had a mass of 2 grams.

a. Find $\int_0^4 R(t)dt$.

b. What does the answer in part a tell you about the size of the crystal at time $t = 4$?

4. (18 points) A circular cone has base radius 6 and height 10. What are the dimensions of the largest (i.e., greatest volume) cylinder which can fit inside it?

Some (not all) of these formulas might be useful:
Volume of a cylinder: $\pi r^2 h$.
Surface area of a cylinder: $2\pi rh + 2\pi r^2$.
Volume of a cone: $\frac{1}{3}\pi r^2 h$.

5. If $F(x) = \int_5^{x^3} \frac{1}{t} dt$, find a formula for $F'(x)$.

6. Sketch a single function $g(x)$ whose first derivative is equal to zero at $x = 0$, $x = 3$, and $x = 10$, but nowhere else, and whose second derivative is negative on the region $(0, 1)$, positive on $(1, 3)$, negative on $(3, 4)$, and positive on $(4, 10)$.