MATH 106 — FIRST EXAM

DEPARTMENT OF MATHEMATICS
Johns Hopkins University

February 25, 2004

NAME: ____________________________ SIGNATURE: ___________________________

TA (circle): Ann Stewart, Grace Currie

1. This exam has five pages including this cover. There are six questions.
2. Use of books, notes, or scratch paper is not allowed. You may certainly use a calculator (but not its manual).
3. **Show all of your work!** Partial credit is available for many problems but can only be given if the graders understand your work. Be sure to explain your reasoning carefully. Include units in your answers whenever appropriate.
4. Read directions carefully. For some problems, a brief answer is sufficient, but others require you to show all work or give explanations.
5. For exam security reasons, students of Dr. Budur *may not leave before 11:05.* We apologize for any inconvenience this may cause.

PROBLEM	POINTS	SCORE
1	20	
2	10	
3	14	
4	18	
5	18	
6	20	
TOTAL	100	

1. (20 points) The function $q(x)$ is graphed below. For each of the items listed, calculate it exactly. If you cannot calculate it exactly, estimate as well as possible.

a. (4 points) $q(7) =$

b. (4 points) $q'(7) =$

c. (4 points) $\lim_{h \to 0} \frac{q(7+h) - q(7)}{h} =$

d. (4 points) The equation of the tangent line to $q(x)$ at $x = 7$.

e. (4 points) $q(7.3)$.

2. (10 points) Calculate the limit. To get full credit, you must explain what you have done and use a reliable, certain method.

$$\lim_{x\to\infty} \frac{x - 3x^3}{(x+1)(x+2)(x+3)(x+4)}$$

3. (14 points) Find a constant c so that the function $f(x)$ defined below is continuous everywhere.

$$f(x) = \begin{cases} \frac{x^2+5x-14}{x-2}, & \text{if } x < 2 \\ 2x - c, & \text{if } x \geq 2 \end{cases}$$

4. (18 points)

a. (6 points) If $g(t) = e^{t\sin(t)}$, find $g'(t)$.

b. (6 points) Differentiate with respect to x: $\frac{\pi x^3+3^e}{\ln(x^2+1)}$.

c. (6 points) Find $\frac{d}{dx}(2x^3 + x)^x$.

5. (18 points) Find the tangent line to the curve $xy - y^3 = 1$ through the point $(2, 1)$.

6. (20 points) Our sun is approximately 1.4 million kilometers in diameter. Reasonable approximations indicate that it is growing (in diameter) at a rate of 7 centimeters per year ($7 \cdot 10^{-5}$ km/year). How fast is the volume of our sun growing? (For a sphere with radius r and volume V, $V = \frac{4}{3}\pi r^3$). (show all work!)