MATH 106 — EXAM 2

Department of Mathematics
Johns Hopkins University

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NAME: ____________________________ SIGNATURE: ___________________________

SECTION NUMBER:_____________

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1. This exam has seven pages including this cover. There are six questions.
2. Use of books, notes, or scratch paper is not allowed. You may certainly use a calculator (but not its manual).
3. **Show all of your work!** Partial credit is available for many problems but can only be given if the graders understand your work. Be sure to explain your reasoning carefully. Include units in your answers whenever appropriate.
4. Read directions carefully. For some problems, a brief answer is sufficient, but others require you to show all work or give explanations.

PROBLEM	POINTS	SCORE
1	12	
2	18	
3	14	
4	20	
5	16	
6	20	
TOTAL	100	

1. (12 points) Circle True or False in each case.

True or False If $f'(c) = 0$, then either c is a local minimum value for f, or else it is a local maximum value for f.

True or False For any continuous function f, $\frac{d}{dx}\int_0^x f(t)dt = f(x) - f(0)$.

True or False $\int_0^x t^5 dt$ refers to a single thing, which is a member of a class of things referred to when we say $\int x^5 dx$.

True or False If $f(x)$ is a continuous function defined on a region $[a, b]$ which has a maximum value at $x = 5$, then 5 must certainly be a place where $f'(x)$ does not exist, a place where $f'(x)$ is equal to zero, or an endpoint a or b.

2. (18 points) Brief answer.

a. (4 points) Write an antiderivative for the function $f(x) = \sin(3x) + \ln(\pi)$.

b. (5 points) Write the derivative of the function $\int_0^x \sin(\sin(\sin(z)))dz$.

c. (6 points) Calculate $\int_1^2 (x^2 + 100)^2 dx$ by hand (you may use calculators for arithmetic, but not for integration). Show work.

d. (3 points) Sketch a function (any crazy function is ok) defined at all points in a closed interval with no maximum value on that interval. Your sketch must be clear enough to illustrate the property.

3. (14 points) Suppose that a single differentiable function $f(x)$ is described by this table of data. Do not infer from apparent trends that $f'(x) = 0$ for other values of x. In fact, assume that all x values for which $f'(x) = 0$ are shown on the table, i.e., that f' is nonzero for x not shown.

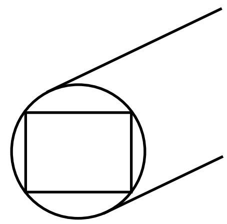
x	-1	2	3	6	9	12
$f'(x)$	0	0	4	0	0	0
$f''(x)$	-1	3	0	0	2	-3

a. (6 points) Based on the data, write a complete list of values of x where a global maximum value of $f(x)$ *might* occur.

b. (3 points) If you think there is anything impossible about the table, identify it and explain. Otherwise just write "ok."

c. (5 points) Calculate $\int_{-1}^{3} f''(x)dx$.

4. (20 points) A rectangular beam will be cut from a cylindrical log of radius 2 feet, so that in cross section, the beam is a rectangle whose corners touch the circumference of a circle (the cross section of the log). The strength of a rectangular beam is proportional to its width, and also to the *square* of its height[1]. How should the beam be cut to make it strongest? Show all work, and be thorough. (i.e., if you can't know for sure based on your work that your answer is right, then it's not going to get full credit).



[1]This is true.

5. (16 points) Calculate the following limits. Show enough work that the graders can tell you know exactly what you're doing.

a. (4 points) $\lim_{x \to 2} \frac{x^4 - x^2 + 3x - 18}{x^2 - 5x + 6}$

b. (4 points) $\lim_{x \to \infty} x^x$

c. (4 points) $\lim_{t \to 0} \ln(t) t^5$

d. (4 points) $\lim_{x \to 0} \frac{x^4}{\cos(x) - 1 + \frac{x^2}{2}}$

6. (20 points) A water treatment plant purifies water today at a rate of ten thousand gallons per day. But the purification rate experiences a continuous exponential growth of 10% per day, due perhaps to population growth.

a. Find a formula for the purification rate as a function of time t, measured in days starting today.

b. Integrate that function, from $t = 0$ to $t = 7$.

c. Explain (briefly but precisely) the meaning of the result in part b.