MATH 106 — EXAM 1

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NAME: _____ SIGNATURE: _____

SECTION NUMBER:_____

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1. This exam has six pages including this cover. There are six questions.
2. Use of books, notes, or scratch paper is not allowed. You may certainly use a calculator (but not its manual).
3. **Show all of your work!** Partial credit is available for many problems but can only be given if the graders understand your work. Be sure to explain your reasoning carefully. Include units in your answers whenever appropriate.
4. Read directions carefully. For some problems, a brief answer is sufficient, but others require you to show all work or give explanations.

PROBLEM	POINTS	SCORE
1	12	
2	14	
3	14	
4	20	
5	20	
6	20	
TOTAL	100	

1. (12 points) Circle True or False in each case.

True or False If for some particular value of x, $f'(x) = 0$, then $f(x)$ is a constant function.

True or False The inverse function $f^{-1}(x)$ is a special name for the function $\frac{1}{f(x)}$.

True or False If the derivative (with respect to time) of food prices is negative, then to save money, you should stock up on lots of nonperishable foods.

True or False If $f(x) = \pi^5$, then $f'(x) = 5\pi^4$

True or False $\lim_{h \to 0} \dfrac{e^{x+h} - e^x}{h} = e^x$.

2. (14 points) Brief answer.

a. Write down a function $f(x)$ so that $\frac{f'(x)}{f(x)} = 3$ for all values of x.

b. Write the derivative of the function $\ln(\sin(\ln(x)))$.

c. Sketch a graph of a function which is always positive, but whose derivative is always negative.

3. (14 points)

a. (4 points) Give the (limit) definition of the derivative $f'(x)$, in terms of $f(x)$. (For no particular choice of $f(x)$ – a generic unknown function.)

b. (10 points) Use this limit definition to find the derivative of x^3. Of course you will get $3x^2$, but it is the work that counts.

4. (20 points) Calculate the following limits. Show enough work that the graders can tell you know exactly what you're doing.

a. (points) $\lim_{x\to\infty} \dfrac{x^2+e^x+1}{3x^3-e^x-4}$

b. (points) $\lim_{x\to 0} f(x)$, where

$$f(x) = \begin{cases} e^x - 3 & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

c. (points) $\lim_{t\to 3} \dfrac{|t-3|}{t-3}$

d. (points) $\lim_{x\to 1} \dfrac{x^4-1}{x^2-1}$

5. (20 points) For this problem you may find it convenient to know that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

A *burl* is an abnormal growth on a tree, caused by injury or infection, and prized by woodworkers for its complex grain structure. Assume that a particular burl is exactly half of a sphere. Assume also that its market price P in dollars is the product of its volume V in cubic centimeters and its age t in years, because older burls are more valued. Our particular burl is 4 years old, has a radius of 5 cm, and is currently growing at 3 cm per year.

a. (8 points) Find an equation relating $\frac{dV}{dt}$ and $\frac{dr}{dt}$, and use it to find $\frac{dV}{dt}$ for this burl.

b. (8 points) Find another equation relating $\frac{dP}{dt}$ and $\frac{dV}{dt}$, and use it to find $\frac{dP}{dt}$.

c. (4 points) Explain what your answer tells us about this burl, in simple (non-calculus) language. (If you have no answer to part b, make one up to answer this part.)

6. (20 points) This problem is about finding the cube root of 210 without a calculator.

a. (8 points) Let $f(x) = x^3$. Find the formula for the tangent line to $f(x)$ at $(6, 216)$. Show all work.

b. (6 points) By hand (not using calculators), use your answer to part **a** to find an approximate solution to the equation $x^3 = 210$. That is, use your answer to part **a** to estimate the cube root of 210. Show all work.

c. (6 points) Now explain whether your answer is close to correct, and if it is close to correct, explain why. Justify convincingly, but please be brief.