MATH 106 — FINAL EXAM

DEPARTMENT OF MATHEMATICS
Johns Hopkins University

December 12, 2003

NAME: ____________________________ SIGNATURE: ___________________________

SECTION NUMBER:______________

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1. This exam has nine pages including this cover. There are twelve questions.
2. Use of books, notes, or scratch paper is not allowed. You may certainly use a calculator (but not its manual).
3. **Show all of your work!** Partial credit is available for many problems but can only be given if the graders understand your work. Be sure to explain your reasoning carefully. Include units in your answers whenever appropriate.
4. Read directions carefully. For some problems, a brief answer is sufficient, but others require you to show all work or give explanations.

PROBLEM	POINTS	SCORE
1	6	
2	8	
3	4	
4	8	
5	8	
6	8	
7	4	
8	10	
9	14	
10	10	
11	10	
12	10	
TOTAL	100	

1. (6 points) State each of the fundamental theorems of Calculus. Please use brief, symbolic, mathematical notation (as opposed to words and phrases) where appropriate.

2. (8 points) To show off your talents as a computer programmer, you have written a program to estimate integrals using the trapezoid rule, and you plan to apply it to approximate the integral $\int_0^3 x^4 dx$. As you can check, the exact value of this integral is 48.6. How many subdivisions of $[0, 3]$ are required to guarantee a trapezoid-rule approximation between 48.5 and 48.7?

3. (4 points) Circle **True** or **False**: The function below is continuous for *all* values of x. (No explanation necessary.)

$$f(x) = \sin(e^{x^2+x+4})\cos(\cos((x-1)(x-2)(x-3)))$$

4. (8 points) Find a constant c so that the function

$$f(x) = \begin{cases} \frac{x}{2e^x-2} & \text{if } x > 0, \\ x + c & \text{if } x \le 0. \end{cases} \qquad (1)$$

is continuous. Show your work carefully.

5. (8 points) Find the equation of the tangent line through the point $(1, e^2)$ to the curve defined by the equation $\ln(xy) = 2x$.

6. (8 points) Compute the Taylor polynomial of degree three about $a = 0$ for the function $f(x) = \sqrt{1+x}$. Show your work.

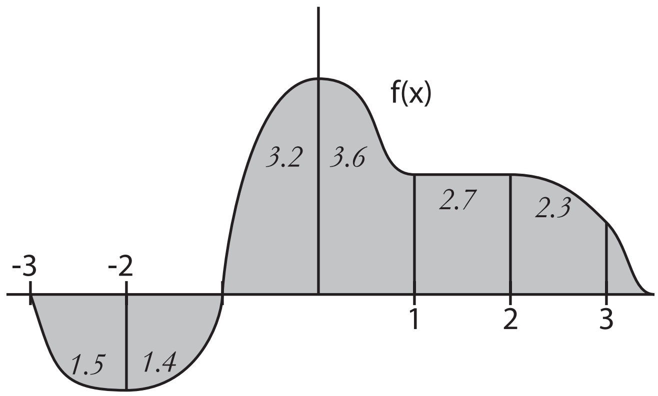
7. (4 points) Simplify:

$$1 + \frac{3}{1} + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \dots$$

8. (10 points) Below is a graph of $f(x)$ with some areas marked in italics. Compute

$$\int_0^2 x f(x^2 - 2) dx$$

as accurately as possible.



9. (14 points) The elite Hopkins Medical Center Sneezologists, through diligent measurement and experimentation, have recorded and modeled the exhalation rate (in cc's per second) during a typical human sneeze. Their approximate formula for the exhalation rate $E(t)$ is

$$E(t) = \frac{1-t}{t^2+2t},$$

where t is time in seconds after the "sneeze initiation event."

a. (10 points) Find the definite integral of this function, from $t = 0$ to $t = 1$.

b. (2 points) Assuming the Sneezologists' formula is correct, interpret your answer to part (a), in one plain English sentence.

c. (2 points) The Sneezologists model is [**Reasonable** or **Unreasonable**]. (Circle one. No explanation necessary)

10. (10 points) Calculate the integral by hand. Of course, show all work.

$$\int e^{2x} \sin(x) dx$$

11. (10 points) Find the volume of a pyramid whose height is h and whose base is a square of side length w. (You must show how to use techniques of integration to solve the problem — a memorized formula for this volume will not be worth credit.)

12. (10 points) If you join a point on the base of a cylinder to an opposite point on the top, you get a line through the center of the cylinder which could be called a “diagonal” (see picture). Many different cylinders have diagonal length 6 inches. Which has the largest volume? (Hints: If you choose wisely, you can avoid ugly square roots in your formulas. Also, $V_{\text{cylinder}} = \pi r^2 h$)