MATH 106 — SECOND EXAM

Department of Mathematics
Johns Hopkins University

November 11, 2003

NAME: _____ SIGNATURE: _____

SECTION NUMBER:_____

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1. This exam has seven pages including this cover. There are six questions.
2. Use of books, notes, or scratch paper is not allowed. You may certainly use a calculator (but not its manual).
3. **Show all of your work!** Partial credit is available for many problems but can only be given if the graders understand your work. Be sure to explain your reasoning carefully. Include units in your answers whenever appropriate.
4. Read directions carefully. For some problems, a brief answer is sufficient, but others require you to show all work or give explanations.

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	16	
4	15	
5	14	
6	15	
TOTAL	100	

1. Three unrelated questions.

a. TRUE or **FALSE:** (4 pts) Every continuous function $f(x)$ has two or more antiderivatives.

(Show no work on True/False. Only the answer is graded!)

b. (8 pts) State each of the fundamental theorems of Calculus. Please use brief, symbolic, mathematical notation (as opposed to words and phrases) where appropriate.

c. (8 pts) Find the approximate value of the sum below, by any method other than adding up all the terms on your calculator! *Show your work to indicate your method.*

$$\frac{1}{1000}\left[\cos\left(\frac{1}{1000}\right)+\cos\left(\frac{2}{1000}\right)+\cos\left(\frac{3}{1000}\right)+\cos\left(\frac{4}{1000}\right)+\ldots+\cos\left(\frac{1000}{1000}\right)\right]$$

2. (20 pts) You are planning your next modern art masterpiece, titled "Red with Frame." You will paint a red rectangle of area 1200 square inches, and attach a frame. The frame will be four inches wide along the sides of the red rectangle, but only three inches wide on the top and bottom. What is the smallest possible area of the whole project, including frame? *Show all work—be thorough!*

3. (16 pts) In this problem, you will "set up" but not completely solve several problems related to one curve with given endpoints. For each part, write one ("grammatically correct") integral representing the quantity sought. You *need not* compute the values of these integrals. There is no need to show any work.

$$\text{The Curve: } y = g(x) = x^3, \quad 1 \leq x \leq 2$$

a. (4 pts) The area under the curve (and above the x-axis).

b. (4 pts) The volume enclosed when the curve is rotated around the x-axis.

c. (4 pts) The volume enclosed when the curve is rotated around the y-axis.

d. (4 pts) The length of the curve.

4. Calculate these limits. Show your work, because credit will not be given for calculator shortcuts.

a. (5 pts) $\lim_{x \to 0} x \ln(x)$

b. (5 pts) $\lim_{x \to \pi} \frac{\sin(x)}{x}$

c. (5 pts) $\lim_{x \to 0} \frac{1}{x}(1 - 2^x)$

5. (14 pts) Sketch a graph of a single function $f(x)$ for which all of the following statements are true:

i. For $x > 2$, $f'(x) > 0$, but for $x < 2$, $f'(x) < 0$.

ii. For $x > 4$, $f''(x) < 0$, but for $x < 4$, $f''(x) > 0$.

iii. $\lim_{x\to\infty} f(x) = 0$.

6. (15 pts) Even after an organism's death, its cells continue to burn oxygen at a decreasing rate. Assume that the rate of oxygen consumption after t hours is $3 - t/4$ grams per hour. How much oxygen is consumed in the three hours immediately following death?