MATH 106 — FIRST EXAM

Department of Mathematics
Johns Hopkins University

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NAME: _____ SIGNATURE: _____

SECTION NUMBER:_____

TA (circle): Ann Stewart, Seok-Min Lee, Heather Angelo, Michael Walsh, Brian MacDonald

1. This exam has six pages including this cover. There are five questions.
2. Use of books, notes, or scratch paper is not allowed. You may certainly use a calculator (but not its manual).
3. **Show all of your work!** Partial credit is available for many problems but can only be given if the graders understand your work. Be sure to explain your reasoning carefully. Include units in your answers whenever appropriate.
4. Read directions carefully. For some problems, a brief answer is sufficient, but others require you to show all work or give explanations.

PROBLEM	POINTS	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

1. Short answer. You need not show work or justify answers for this page.

a. TRUE or **FALSE:** Every exponential function has a constant per capita growth rate.

b. TRUE or **FALSE:** Because $\frac{dx}{dy} = \frac{1}{dy/dx}$, the derivative of $\arcsin(x)$ is $\frac{1}{\cos(x)}$.

c. TRUE or **FALSE:** A function defined on a closed interval must have a global max on that interval.

d. Differentiate $\frac{x\sin(x)}{g(x)} + e^{\pi}$ with respect to x. Here $g(x)$ is an unknown function.

e. Calculate $\lim_{x\to\infty} \frac{3-x^2}{5x^2-2x+e^{-x}}$

f. Let E be the total yearly advertising expenditure (in dollars) promoting cigarette use in the USA, and let L be the average USA life expectancy, in years. Assume L is a function of E. Guess a numerical value for $\frac{dL}{dE}$. Of course you can't be precise here–just write down something reasonable. Include units.

2. A woman standing still on a lakeside pier pulls a raft into place by collecting a rope attached to the raft. Assume that the rope is connected to the raft at water level, that the point at which she holds the rope is five feet above water level, and that the rope is straight. The woman silently notes that 20 feet of rope separate her from the raft, and that she is gathering 2 feet of rope per second. How fast is the raft moving? (You must show your work for full credit.)

3. The stride length[1] s (in meters) and the velocity v (in meters per second) of an ostrich are related by the equation:

$$v^2 + v = s^3$$

a. A certain ostrich is observed running 14 meters per second. Calculate $\frac{dv}{ds}$ for this bird.

b. Explain in simple, nontechnical language the meaning of the answer to part a. Be concise—one sentence should be enough. (If you can't do part a, you may use a made-up answer to complete part b.)

[1]Stride length means the distance between successive footprints

4. In this problem we will examine the function $x^4 + x - 15$.

a. Find the equation of the tangent line to this function at the point $x = 2$.

b. Using the answer from part a, find an approximate solution to the equation

$$x^4 + x - 15 = 0.$$

You need not find an exact solution.

5. *Using the definition of the derivative,* calculate the derivative of $f(x) = \frac{1}{x}$. Your response will be judged not by the answer on the last line, but by the thoroughness and correctness of the step-by-step explanation.