A note on Brown-Peterson cohomology from Morava K-theory I,II by Ravenel, Wilson, and Yagita, and by Wilson with correction to several papers

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1 Main Result

We use a convention from [4] in this note so that $P(0)$ will denote p-completed Brown-Peterson cohomology. In papers [4, 5], one can find the following.

Theorem 1 (Theorem 1.17 of [4] and Theorem 1.2 of [5]) *Let X_i be spaces of finite type such that $P(k)^*(X_2)$ is Landweber-flat. Suppose that the map $f : X_1 \to X_2$ induces surjection in $K(n)^*(-)$ for all $n \geq k, n > 0$. Then $P(k)^*(f)$ is surjective.*

Here, the word *surjective* signifies an epimorphism in an appropriate topologized categoy. This is because the proof relies on Theorems 1.20 1nd 1.21 of [4], which gives a set that generates *topologically* $P(k)^*(X)$.

On the other hand, in works [1, 2, 3] the above theorem was quoted incorrectly, that is the word *surjective* was interpreted as surjective on underlying sets. Thus, as is, the results in [1, 2, 3] are compromised.

The purpose of this note is to show that the above theorem still holds with the interpretation in works [1, 2, 3].

This can be seen as follows. First recall that the category of profinite abelian groups is abelian, since it is Pontryagin dual to the category of torsion abelian groups, which is abelian. This implies that if there is a continuous homomorphism between two profinite abelain groups. More precisely, let $B \to C$ be an epimorphism in the category of profinite abelian groups. Then we have a short exact sequence $A \to B \to C$ in the category of profinite abelian groups. Denote by $M' \cong Hom_c(M, S^1)$, the set of continuous homomorphism from M to S^1. Then we have a short exact sequence $C' \to B' \to A'$. Now, since we have $A \cong Hom(A', Q/Z)$ etc, by the injectivity of Q/Z, the map $B \to C$ is surjective in the category of abelian groups. By Remark 1.6 of [4] we have $\varprojlim P(k)^*(sk_m X) \cong P(k)^*(X)$ for all k. Thus when $k > 0$, the usual skeltal topology on $P(k)^*(X)$ is profinite. When $k = 0$, instead of the traditional skeltal topology, one can use the p-adic skeltal topology, that is, one can topologize $P(0)^*(X)$ as the inverse limit of $P(0)^*(sk_m X)/p^l$. Now, $P(0)^*(X_i)$ are clearly profinite, and since this topology is clearly coarser than the skeltal topology, the image of $P(0)^*(f)$ is also dense in p-adic skeltal topology. This means $P(0)^*(f)$ is an epimorphism of profinite abelain groups, so it is a surjection of underlying sets.

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