FINAL FOR 110 405
FALL 2003

Answer all six questions. The first two questions are True/False. Fully justify your answer for the last four questions.

Question 1. (10 points; True/False). A function which is differentiable everywhere must also be continuous.

Question 2. (10 points; True/False). The continuous image of a closed set is always closed.

Question 3. (20 points). One of our theorems said that a continuous function on a compact set must be uniformly continuous. Prove this.

Question 4. (20 points). Suppose that f is a continuous function defined on an interval. Show that the image is also an interval.

Question 5. (20 points). Suppose that $f : \mathbf{R} \to \mathbf{R}$ satisfies $f \geq 0$ and $f'' \leq 0$ everywhere. Show that f must be constant.

Question 6. (20 points). Suppose that f is a positive, continuous function on $[0, 1]$. First (10 points), prove that for any $\lambda > 0$ we have

$$2 \int_0^1 f \leq \lambda^2 + \lambda^{-2} \int_0^1 f^2 .$$

Second (the other 10 points), show that

$$\int_0^1 f \leq \left(\int_0^1 f^2 \right)^{1/2} .$$