Math 104 **Fall 2009**

Homework 2

Due Wednesday, October 7, 2009

1. Let A and B be square, invertible matrices. Prove that the inverse of AB is $B^{-1}A^{-1}$.
2. Exercise 1.3, Chapter 1 of Trefethen-Bau.
3. (a) Find a square matrix A, whose entries are not all zeros, such that $A^2 = 0$. (The matrix A^2 is of course AA.)
   (b) Exhibit a nonzero vector that belongs to the nullspace of the matrix you just constructed.
   (c) In general, prove that if a matrix B satisfies $B^2 = 0$, then it cannot be invertible.
4. If u and v are two vectors such that $\|u\| = 3$ and $\|v\| = 5$,
   (a) what are the smallest and largest values of $\|u - v\|$?
   (b) and what are the smallest and largest values of $\langle u, v\rangle$?
5. Exercise 2.1, Chapter 2 of Trefethen-Bau. [A diagonal matrix is a matrix whose off-diagonal elements ($i \neq j$) are zero.]