**Math 104** **Fall 2009**

# Homework 1

Due Wednesday, September 30, 2009

Please write neat and complete solutions to the problem sets. "Neat" means well structured, both esthetically and logically. "Complete" means that the grader will need to see a sufficient amount of explanations and details to give you full credit, even if the question only asks for a numerical answer. Thanks.

1. Consider the three vectors

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \qquad v_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \qquad v_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}.$$

   (a) Find the dimension of the vector space generated by $v_1, v_2, v_3$.

   (b) Do $v_1, v_2, v_3$ form a basis for the space they generate?

   (c) Find a matrix, whose entries are not all zeros, and whose nullspace contains all three vectors $v_1, v_2, v_3$.

   (d) Find a vector $w$ such that $\{v_1, v_2, w\}$ is a basis for $\mathbb{R}^3$.

2. Justify the statement: if a collection of vectors contains the zero vector, then there is no chance that the vectors in the collection be linearly independent.

3. Two vectors are said to be collinear when one can be written as a scalar multiple of the other. Consider two vectors $u$ and $v$ that are not collinear. Consider a vector $w$ that does not belong to the linear span of $u$ and $v$. Prove that $u, v, w$ are linearly independent.

4. A matrix is said to be upper triangular if $a_{ij} = 0$ for $i > j$. Consider a generic 3-by-3 upper triangular matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}.$$

   (a) If $a_{11}, a_{22}$, and $a_{33}$ are nonzero, show that the only solution to $Ax = 0$ is $x = 0$.

   (b) If either $a_{11} = 0$, or $a_{22} = 0$ or $a_{33} = 0$, then prove that the columns are linearly dependent. (Consider all three cases separately.)

   (c) If $a_{22} = 0$, find a nonzero element in the nullspace of $A$.