Math 104 : Midterm

Instructions: Complete the following 4 problems. Remember to show all your work. No notes or calculators are allowed. Please sign below to indicate you accept the honor code.

Name: ______________________________

SUID: ______________________________

Signature: ______________________________

Problem	1	2	3	4	Total
Score					

Problem #1. (20 pts) Let $\mathbf{w}_1$, $\mathbf{w}_2$ and $\mathbf{w}_3$ be three vectors in $\mathbb{C}^3$. Let

$$\begin{aligned}
\mathbf{v}_1 &= \mathbf{w}_1 - \mathbf{w}_3, \\
\mathbf{v}_2 &= \mathbf{w}_1 + \mathbf{w}_2, \\
\mathbf{v}_3 &= \mathbf{w}_1 + \lambda \mathbf{w}_3, \text{ and} \\
\mathbf{v}_4 &= 2\mathbf{w}_1 + \mathbf{w}_2 - \mathbf{w}_3.
\end{aligned}$$

Where here $\lambda \in \mathbb{C}$. For what value λ_0 is it always true that when $\lambda = \lambda_0$, $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ and $\mathbf{v}_4$ never span $\mathbb{C}^3$. Justify your answer. (Hint: Rewrite the problem using matrices).

Problem #2. (30 pts) Let

$$\mathbf{v} = \begin{bmatrix} 2\sin\theta \\ -2\cos\theta \end{bmatrix} \in \mathbb{R}^2$$

Let $A \in \mathbb{R}^{2\times 2}$ denote the matrix which gives orthogonal projection onto $span(\mathbf{v})$.

a) Determine A.

b) Determine $N(A)$ and $R(A)$.

c) Determine a full QR factorization of A.

Problem #3. (20 pts) Let $A, B \in \mathbb{C}^{m \times m}$ suppose that $AB = 0$ and $BA = 0$.

a) What, if any, is the relationship between the null space of A and the column space of B? Justify your answer.

b) Show that either $\dim N(A) \geq \frac{m}{2}$ or $\dim N(B) \geq \frac{m}{2}$.

Problem #4. (30 pts)

a) Suppose that $A, B \in \mathbb{C}^{m \times m}$ are unitary matrices. Verify that A^* and AB are also unitary. (Hint: Use the algebraic properties of the adjoint)

b) Let

$$\mathbf{v}_1 = \frac{1}{5}\begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix}, \mathbf{v}_2 = \frac{1}{5}\begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \in \mathbb{R}^3$$

Verify that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthonormal basis of $\mathbb{R}^3$. Justify your answer.

c) Let

$$\mathbf{w}_1 = \frac{\sqrt{2}}{2}\begin{bmatrix}1\\0\\1\end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix}0\\-1\\0\end{bmatrix}, \mathbf{w}_3 = \frac{\sqrt{2}}{2}\begin{bmatrix}-1\\0\\1\end{bmatrix} \in \mathbb{R}^3$$

be a set of orthonormal vectors. Determine the orthogonal matrix $U \in \mathbb{R}^{3\times 3}$ so that $U\mathbf{v}_i = \mathbf{w}_i$ for $i = 1, 2, 3$ here the $\mathbf{v}_i$ are given in b). (Hint: Use part a))