\[ \nabla f(x, y) = \begin{bmatrix} 2x + 2 \\ 2y \end{bmatrix} \]
\[ \nabla f(0, 0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \]
\[ \sqrt{1^2 + 2^2} = \sqrt{5} \]
\[ \nabla f(0, 0) \cdot \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{\sqrt{5}} (2 \cdot 1 + 0 \cdot 2) = \frac{2}{\sqrt{5}} \]
\[ z - f(x_0, y_0) = \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0) \]
\[ z - 1 = 2x \]
\[ f(0, 0) = 1 \]
\[ f(x, y) = 1 \]
\[(x + 1)^2 + y^2 = 1 \text{ is a circle centered at } (-1, 0) \text{ with radius 1.} \]
\[ \frac{\partial f}{\partial x} = 2x + 2 = 0 \text{ and } \frac{\partial f}{\partial y} = 2y = 0 \implies x = -1 \text{ and } y = 0. \text{ Thus there's a critical point at } (-1, 0). \]

The Hessian of \( f \) is \( \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \).

The boundary of \( D \) is given by \( p(t) = (2 \cos \theta, 2 \sin \theta) \).

Method 1:
\[ F(\theta) = f(2 \cos \theta, 2 \sin \theta) = 4 + 4 \cos \theta + 1 = 5 + 4 \cos \theta \]
Then \( \frac{dF}{d\theta} = -4 \sin \theta = 0 \). Then \( \theta = k\pi \) for any integer \( k \). These solutions correspond to the points \((-2, 0)\) and \((2, 0)\).

We compare the values at the critical points:
\[ f(-1, 0) = 0 \]
\[ f(-2, 0) = 1 \]
\[ f(2, 0) = 9 \]

We conclude \( f \) has an absolute minimum at \((-1, 0)\) and an absolute maximum at \((2, 0)\).

Method 2:
\[ g(x, y) = x^2 + y^2 - 4 \]
\[ \nabla f = \lambda \nabla g \text{ gives the equations} \]
\[ 2x + 2 = \lambda 2x \text{ and } 2y = \lambda 2y \]

There are different ways to solve this.

One way gives \( 2xy + 2y = \lambda 2xy = 2xy \). Then \( y = 0 \). Then using the constraint \( x^2 + y^2 = 4 \), we conclude \( x = \pm 2 \). We complete the problem as in Method 1.

Intuitively, we look for the furthest point in \( S \) away from the center \((-1, 0)\) of the paraboloid. This occurs equally at two of the corners: \((4, 4)\) and \((4, -4)\).