Homework 9 Solutions (Section 12.1)

December 2, 2013

0.1 Exercise 2

Three levels of leaf damage and four different watering protocols. For each combination of watering protocol, you plan to have three replicates. How many replicates total?

Because choosing leaf damage and water protocol are distinct, it doesn’t matter if we choose the leaf damage level before or after watering protocols. However, we must choose both of these before counting replicates. For leaf-protocol combinations we’ll have

$$3 \cdot 4 = 12$$

combinations. Thus there are

$$12 \cdot 3 = 36$$

replicates total.

0.2 Exercise 6

Two sexes. Three choices of food. Twelve rats for each food-and-sex combination. How many rats total?

Similar to the last problem, we’ll have

$$2 \cdot 3 \cdot 12 = 72$$

rats total.

0.3 Exercise 10

Five people line up. How many different lineups are possible?

The key here is that there is an order to which the people line-up. First someone comes in line, that could have been 5 choices here. Then choose a second person from among 4 people. And so on. Thus, there are

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120$$

possible lineups.
0.4 Exercise 16
Play four different songs out of ten different songs. How possibilities are there?
Again, there is an order to the songs being played, but we only play four of them. Thus we have

\[ 10 \cdot 9 \cdot 8 \cdot 7 = 10 \cdot 24 \cdot 21 = 10(480 + 24) = 5040 \]

possibilities.

0.5 Exercise 24
Twelve people. Five go into an elevator. How many possibilities?
It doesn't matter who went into the elevator first, second, etc.. So if we count with ordering, we have

\[ 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \]
and then eliminate the ordering by dividing by

\[ 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1. \]
More succinctly, there are

\[ \binom{12}{5} = 792 \]
possible ways to choose five out of twelve people (that's why we read \( \binom{12}{5} \) as “twelve choose five”).

0.6 Exercise 28
Three hundred books. Molly buys five. How many possibilities?
Well, there are three hundred and we choose five, so there must be

\[ \binom{300}{5} \]
(three hundred choose five) possibilities.

0.7 Exercise 30
Twelve children. Group of three, group of four, and group of five. We don't care about order within a group.
0.7.1 Method 1

Note that the groups are of different sizes and that sets them apart from one another. Thus we may impose an order to which we choose the groups. Let’s choose the group of three first. There are
\[
\binom{12}{3}
\]
possibilities. We are left with nine children. Let’s choose the group of four. There are
\[
\binom{9}{4}
\]
possibilities. We are left with five children. Let’s choose the group of five. There are
\[
\binom{5}{5} = 1
\]
possibility. Thus in total we have
\[
\frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} \cdot \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 27720
\]
possibilities.

0.7.2 Method 2

Of course, there are many ways to count. We could have also lined up all twelve children for 12! possibilities. Then count off the first three, four, and then five. Once they’ve been grouped, we no longer care about their order, so we divide by 3!, 4!, and 5!. Getting the same answer as above, we have
\[
\frac{12!}{3!4!5!} = 27720
\]
possibilities.

0.8 Exercise 32

Three different variety of beans with fifteen of each variety in a bag.

1. How many ways to choose three of each variety.

2. How many ways only one variety appears.

Solution to 1. For a given variety, we’d have
\[
\binom{15}{3}
\]
ways to choose five out of fifteen beans. Each variety is different, so we can multiply these possibilities together.
total possibilities.

Solution to 2. First we choose the variety that appears. Then find the number of ways for that variety to appear in all nine beans. Thus we have

$$3 \cdot \binom{15}{3}$$

total possibilities.

0.9 Exercise 38

Two different* committees of three people each from a group of nine.

1. No person serves on more than one.

2. People can serve on both.

*There’s a slight ambiguity to whether the committees are different. Due to part 2, that people can serve on both, it makes sense to assume that is the case. Don’t be afraid to clarify such ambiguities.

Solution to 1. Because the committees are different, this exercise is similar to Exercise 30. We have

$$\binom{9}{3} \cdot \binom{6}{3}$$

total possibilities if no person can serve on more than one. Note that you can also use the second method of Exercise 30. Except here, there’s people remaining, think of them as being in the group of unchosen people. So we have

$$\frac{9!}{3!3!3!}$$

total possibilities. One can check this gives the same answer.

Solution to 2. If people can serve on both, then the pool of people to choose from is the same for each committee and we have

$$\binom{9}{3} \cdot \binom{9}{3}$$

total possibilities.

0.10 Exercise 40

Humans have 23 pairs of chromosomes. How many possible gametes produced by a human?

Each gamete takes one of the two chromosomes from each pair and each pair is different. Thus we might as well order the pairs and count each task
separately. Then multiply those possibilities together. In other words, fix a chromosome number and then from two choose one. We have

\[
\binom{2}{1} = 2
\]

possibilities per chromosome number. In total, there are

\[
2 \times \ldots \times 2 = 2^{23}
\]

23 times possible gametes produced. (That’s 8,388,608 gametes. Note that given two people, that’s \(2^{46} \approx 7.03 \times 10^{13}\) different possible humans produced.)