1. The least common multiple of 1, 2, 3, 4, and 5 is 60, so we are looking for the number of three-digit multiples of 60. Since $1000/60 = 16\frac{2}{3}$, giving 16 multiples of 60 under 1000, and 60 has 2 digits, the answer is $16 - 1 = 15$.

2. The number of ways to choose two fish is $\binom{16}{2} = 120$. To choose two fish of opposite gender we need one male fish (6 choices) and one female fish (10 choices), giving a total of 60 choices, so the answer is $\frac{60}{120} = \frac{1}{2}$.

3. There are 13 odd and 12 even numbers. The max of 13 and 12 is 13, so choosing 14 numbers will guarantee at least one of each.

4. Most numbers have an even number of factors, since they come in pairs that multiply to the number $n$: i.e. the pair $k$ and $\frac{n}{k}$. Squares are the exception with an odd number of factors, since the pair of identical numbers $\sqrt{n}$ and $\sqrt{n}$ multiply to $n$. If a square has 7 factors, the fourth factor happens to be the middle one, $\sqrt{n}$. Thus $n = 8^2 = 64$.

5. **Solution 1:** The number of ways to roll the first die is 6, the second die 5 (can’t match the first), and the third 4 (can’t match the first two), giving 6·5·4 = 120 ways. The number of ways to number of ways to roll the first die with no 4’s is 5, the second die 4, and the third die 3, for a total of 5·4·3 = 60 ways. The desired probability is the complement of the probability that you obtain no 4’s (since the numbers are distinct), so the probability is $1 - \frac{60}{120} = \frac{1}{2}$.

**Solution 2:** The three die give three distinct numbers. Since there are 6 numbers to choose from, the probability that any number comes up is $\frac{3}{6} = \frac{1}{2}$.

6. Let the two numbers be $a$ and $b$. Since $a + b = 4x$ and $a - b = 4y$, $x$ and $y$ integers, we get $a = 2(x + y)$ and $b = 2(x - y)$. Thus, $a$ and $b$ are two even numbers that differ by a multiple of 4. There are 6 such pairs $[(0,4), (0,8), (4,8), (2,6), (2,10), (6,10)]$ with $\binom{11}{2} = 55$ total pairs, so the answer is $\frac{6}{55}$.

7. Each person can have four possible outcomes: $HH, HT, TH,$ and $TT$, so he flips one head and one tail $\frac{1}{2}$ of the time (call this outcome $D$) and two heads/two tails $\frac{1}{2}$ of the time (call this outcome $S$). The three people can have eight outcomes: $DDD, DDS, DSD, SSD, SDD, SDS, DSS$, and $SSS$. Of these, $DDS, DSD,$ and $SDD$ satisfy the requirements, and the answer is $\frac{3}{8}$. 
8. Let $C_1$ be the scenario that juror 1 decides correctly, $I_1$ be the scenario that he/she decides incorrectly, and define $C_2, I_2, C_3,$ and $I_3$ similarly. For the jury to decide correctly, we need either $C_1 \cdot C_2 \cdot I_3, C_1 \cdot I_2 \cdot C_3, I_1 \cdot C_2 \cdot C_3,$ or $C_1 \cdot C_2 \cdot C_3$. The corresponding probabilities are $p \cdot p \cdot \frac{1}{2}, p \cdot (1 - p) \cdot \frac{1}{2}, (1 - p) \cdot p \cdot \frac{1}{2},$ and $p \cdot p \cdot \frac{1}{2}$. Summing these gives $p$.

9. To roll exactly one six, two of the three dice need to be 1, 2, 3, 4, or 5 and the other die 6. Since the die with the 6 can be any of the three, the number of ways to do this is $5 \cdot 5 \cdot 3 = 75$; to roll exactly two sixes, one of the three dice need to be 1, 2, 3, 4, or 5 and the other two dice 6. Since the non-6 die can be any of the three, the number of ways to do this is $5 \cdot 3 = 25$; finally, there is only one way the roll three sixes. Since there are $6^3 = 216$ total ways to roll three dice, the expected winning is the probability of each happening weighted by the monetary prize for each. This equals $\frac{75}{216} \cdot \$2 + \frac{25}{216} \cdot \$4 + \frac{1}{216} \cdot \$6 = \$1$.

10. Let $p_b$ be the probability of drawing a quarter at first and $p_a$ be the probability of drawing a quarter at the end, so that $p_b = q$ and $p_a = 1 - q$. Note that the difference between the number of dimes and the number of quarters never changes. Thus, if $p_b > \frac{1}{2}$ then $p_a > \frac{1}{2}$, and if $p_b < \frac{1}{2}$ then $p_a < \frac{1}{2}$. However, since $p_b + p_a = 1$, these scenarios are impossible, so the only possibility is $p_b = q = \frac{1}{2}$. 