110.109 CALCULUS II
SPRING 2015
PRE-MIDTERM II PRACTICE (SET 2)

Name: ______________________

Recitation section:

  1. Tuesday 3:00 (D. Ginsberg)
  2. Tuesday 4:30 (D. Ginsberg)
  3. Thursday 1:30 (P.Y. Chang)
  4. Thursday 3:00 (M. Farag)
  5. Thursday 3:00 (D. Seitova)

Work quickly and carefully, and write your solutions clearly. For clarity, it is recommended that you put
your final answer in a box. However, please show your work and cite all the theorems or lemmas wherever
applicable; partial credit will be given. Keep your cool and manage the time well.

Statement of ethics
I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ______________________ Date: ________________

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<th>Problem</th>
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Problem 1 (10 points). Evaluate the integral, if it exists

\[ \int_{0}^{1} \frac{\ln x}{\sqrt{x}} \, dx \]
Problem 2 (2 × 5 = 10 points).

- Find the slope of the tangent to the curve $r = 3 + \cos 3\theta$ at $\theta = \frac{\pi}{2}$

- Find the exact length of the polar curve $r = 2 \cos \theta$, $0 \leq \theta \leq \pi$
Problem 3 \( (4 \times 5 = 20 \text{ points}) \).

Determine whether or not the sequence \( a_n \) converges, and find the limit if it does converge.

- (a) \( a_n = \frac{8n - 7}{7n - 8} \)

- (b) \( a_n = 10 - (0.99)^n \)

- (c) \( a_n = \frac{(\ln n)^3}{n^2} \)

- (d) \( a_n = \frac{n - e^n}{n + e^n} \)
Problem 4 (2 × 5 = 10 points).
Determine whether the following series are convergent or divergent. Explain your reasoning clearly.

• (a) \( \sum_{n=1}^{\infty} \ln \frac{n^2 + 1}{2n^2 + 1} \)

• (b) \( \sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2} \)