Name: ____________________________________________

Recitation section:

  1. Tuesday 3:00 (D. Ginsberg)  
  2. Tuesday 4:30 (D. Ginsberg)  
  3. Thursday 1:30 (P.Y. Chang)  
  4. Thursday 3:00 (M. Farag)  
  5. Thursday 3:00 (D. Seitova)

Work quickly and carefully, and write your solutions clearly. For clarity, it is recommended that you put your final answer in a box. However, please show your work and cite all the theorems or lemmas wherever applicable; partial credit will be given. Keep your cool and manage the time well.

**Statement of ethics**
I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ____________________________________________ Date: __________________

<table>
<thead>
<tr>
<th>Problem</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>/10</td>
</tr>
<tr>
<td>2</td>
<td>/10</td>
</tr>
<tr>
<td>3</td>
<td>/15</td>
</tr>
<tr>
<td>4</td>
<td>/10</td>
</tr>
<tr>
<td>5</td>
<td>/5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>/50</td>
</tr>
</tbody>
</table>
Problem 1 (10 points). Solve the differential equation

\[ y' + y \ln x = \frac{1}{x^x}, \quad x > 0. \]

Hint: Rewrite \( \frac{1}{x^x} \) in terms of \( e^{(something)} \).
Problem 2 (2 × 5 = 10 points).
Compute the following integrals
• (A) \[ \int e^x \cos x \, dx \]
• (B) \[ \int \sin^3 x \cos^2 x \, dx \]
Problem 3 (15 points). Using integration by parts, prove the reduction formula

\[
\int \frac{1}{(1 + x^2)^n} \, dx = \frac{x}{2(n-1)(1 + x^2)^{(n-1)}} + \frac{2n - 3}{2(n-1)} \int \frac{1}{(1 + x^2)^{(n-1)}} \, dx
\]
**Problem 4** (10 points). Find the general solution of the differential equation

\[ y' + \sin\left(\frac{x + y}{2}\right) = \sin\left(\frac{x - y}{2}\right). \]

Hint: The following formulas could be useful

\[
\sin(A + B) = \sin A \cos B + \cos A \sin B \\
\sin(A - B) = \sin A \cos B - \cos A \sin B.
\]
Problem 5 (5 points). Describe the motion of a particle with position \((x, y)\) as \(t\) varies in the given interval

\[ x = 3 + 2 \cos t, \quad y = 1 + 2 \sin t, \quad t \in \left[ \frac{\pi}{2}, \frac{3\pi}{2} \right] \]