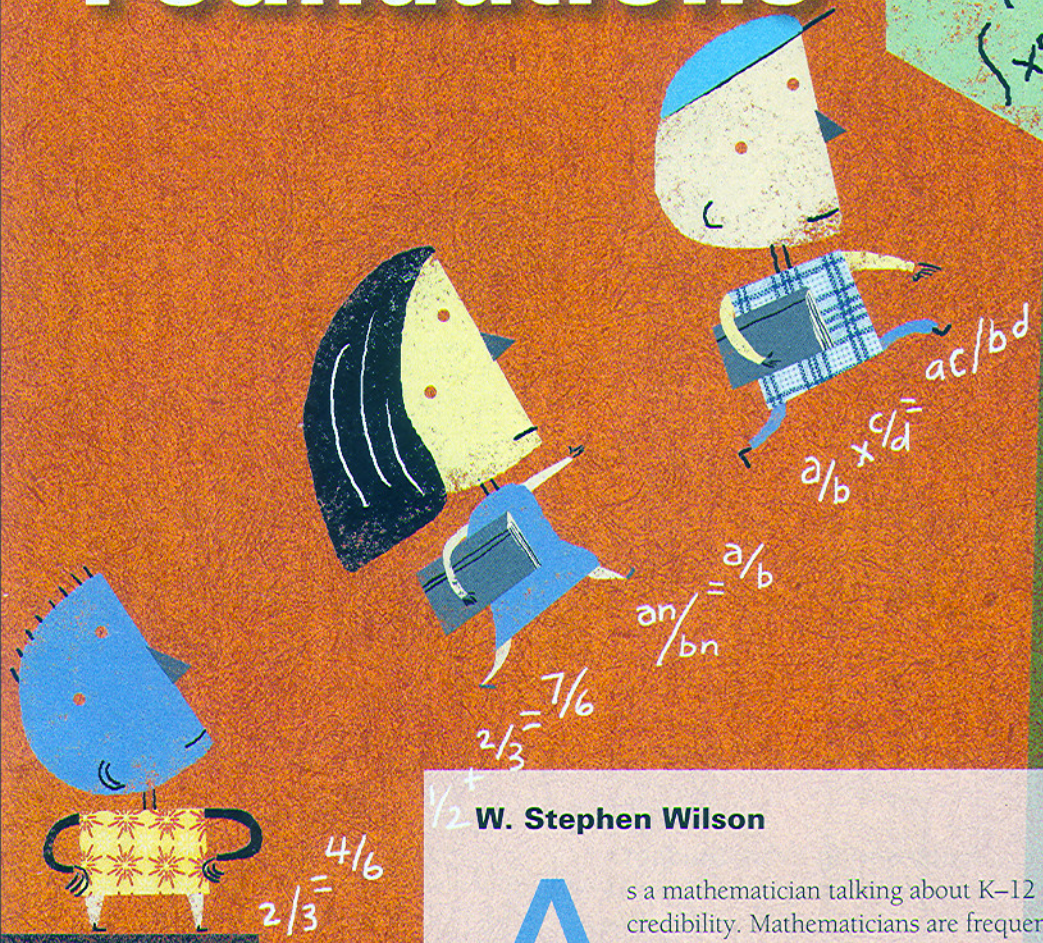
In Defense of Mathematical Foundations



Math standards tend to shortchange arithmetic. But a mastery of arithmetic is just what students need to succeed in higher-level math.

W. Stephen Wilson

As a mathematician talking about K–12 mathematics, I need to establish some credibility. Mathematicians are frequently dismissed because they never taught K–12 math or because all they supposedly care about is creating more mathematicians. However, most of our students are first-year college students, making us 13th grade teachers.

What is surprising is that most of what we college math professors teach isn't college math at all. According to the most recent Conference Board of Mathematical Sciences survey, approximately 3 million students were taking mathematics courses in college (Lutzer, Rodi, Kirkman, & Maxwell, 2007). Most people who follow education are familiar with the fact that roughly 39 percent of these students were in remedial classes. What most people don't realize is that another 34 percent were taking high school mathematics classes for college credit, such as the renamed College Algebra. Thus, 73 percent of the mathematics taught in colleges across the United States is really K–12 mathematics. Students who do not learn this mathematics in high school rarely go into careers that

require real college-level mathematics.

I didn't get involved with K–12 mathematics education just because I think that most students aren't prepared for college math. I got involved because my son was in a school in which the principal and the head of the mathematics department told parents to their faces that long division was obsolete, that the school would never teach it, and that parents who thought the school should teach it were dinosaurs. Of course, one of those parents taught his kid long division anyway; that kid instantly became the most popular kid on the baseball team because he was the only one who knew how to compute batting averages (a rare opportunity for graft and corruption in mathematics).

This raises the questions, Is long division obsolete? Is arithmetic irrelevant? Before I make the case for the dinosaurs, let me present the side of the anti-arithmetic crowd.

A Disturbing Disconnect

Recently, with Gabrielle Martino, I read and reviewed 52 sets of K–12 mathematics standards—for the 50 U.S. states, the District of Columbia, and the Common Core State Standards Initiative (Carmichael, Martino, Porter-Magee, & Wilson, 2010). It was an arduous task; one of the major benefits of so many states adopting the Common Core State Standards is that no one will ever have to review all those individual state standards again.

The big picture I took away from this experience is the complete disconnect between elementary school math and college math requirements. The majority of states fail to focus on the mathematics that elementary school children need to learn to be successful in college math. And that's arithmetic—understanding and fluency with the standard procedures for addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals, along with working complex multistep problems involving operations. It almost

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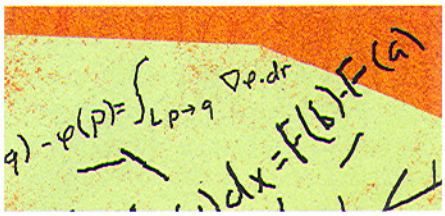

seemed like a plot to prevent children from leaving home for college or, at least, to get them to come home quickly because of lack of preparedness.

Some data: Twenty-one sets of state standards make it clear that arithmetic is a low priority. Their elementary school standards are full of flips, slides, and turns—terms in geometry that refer to transformations of two-dimensional shapes, such as "flipping" a shape across a line so that it faces in the opposite direction—as well as lots of statistics and probability. Arithmetic be damned. Eighteen sets of standards make arithmetic a clear priority—but making something a priority doesn't necessarily mean you do it right. Only 12 of the 52 sets of standards require quick recall of the multiplication tables. Fifteen states make it clear that they don't require such recall, and another 25 states manage to be ambiguous about it.

Only seven states explicitly expect students to know the standard algorithm for whole number multiplication. Twenty-four states consciously undermine the standard algorithms by, for example, offering alternative algorithms or supporting student-invented algorithms. Students are frequently told to use the approach they are most comfortable with; a choice made by a 10-year-old might not be a good choice for a college student. It's as though the writers of those standards went and talked to college mathematics teachers, found out exactly what students need for a solid foundation in math, and then deliberately set about making sure that students *aren't* prepared for college math.

Let me say something about calculators here. Only 20 states have purged calculators from their elementary school standards. Many others give calculators the same status that they give pencil and paper. I've never met a mathematics concept I needed a calculator to teach or assess. High schools run rampant with graphing calculators, but that's because the advanced placement (AP) calculus tests are now mostly about pushing buttons. As a result, we at Johns Hopkins University now find that a 4 on either of the AP calculus tests is inadequate preparation for our next calculus course; starting next year, we'll require that students have a 5 on the AP Calculus AB exam to place out of Calculus I and a 5 on the AP Calculus BC exam to place out of Calculus II.

Although a majority of states don't think arithmetic is all that important—most elementary school teachers know better—most college teachers give it high ratings. A 2009 ACT survey asked teachers to rank more than 100 items. College teachers gave one of the highest rankings—higher than even middle or high school teachers gave—to "Performing addition, subtraction, multiplication, and division on signed rational numbers." (For all you non-mathematicians out there, *signed rational numbers* just means positive and negative fractions.)

On a more personal note, I polled members of an electronic mailing list I belong to, asking them whether they

agreed or disagreed with the following statement: "To succeed at freshman mathematics at my college/university, it is important to have knowledge of and facility with basic arithmetic algorithms—for example, multiplication, division, fractions, decimals, and algebra (without having to rely on a calculator)." I got 93 responses (we're a small community), and they all agreed with the statement.

One ex-dean wrote, "That it is even slightly in doubt is strong evidence of very distorted curriculum decisions." A professor at the National University of Singapore wrote, "Without such facility, no one gets to enter the university." (This comment calls for a cultural aside. Even though Johns Hopkins is a selective university, we have no way of knowing whether our entering students know arithmetic because both the SAT and ACT tests allow the use of calculators.) From Germany, "What a question! The answer is, of course, yes!" From Japan, "I thought it was a joke for you to have asked our opinion about such a self-evident truth."

It's Elementary

Should elementary schools stand on their state standards and give students a weak preparation in arithmetic over the objections of college teachers? Wouldn't it make more sense to listen to the mathematicians who, in time, will get those students in their classrooms?

In a humble attempt to get people to do research on the subject, I gave an arithmetic test to my 200-plus Calculus III students on the first day of class. These were mostly freshmen who had taken the AP calculus test and received credit for a full year of calculus. There were only 10 questions, and the results are revealing. Of the students who missed 3 or more questions, almost 50 percent either dropped the course or did badly on the final exam. Of the 33 students who were clueless about how to do the long division problem, 11 ended up on probation in later years.

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(There's only a 1 in 100 chance of that happening at random.)

Students don't really add, subtract, multiply, and divide a lot in advanced mathematics courses, so why do we mathematicians insist on this? If you're going to study mathematics in college, you *have* to learn to study it in K–12—and that starts in elementary school.

Like it or not, arithmetic is the foundation of mathematics. Studying slides, flips, and turns year after year (as many state standards stipulate) will not prepare a student for college mathematics, nor will related problems appear on the college placement test that students have to take. Students must study arithmetic. The standard algorithms for whole numbers are the only really big theorems that students can be taught in elementary school. It is deep, beautiful, and powerful mathematics. Master these algorithms with understanding, and you're ready to go.

Math standards have also failed to recognize the importance of fractions. Fractions are core to mathematics, yet there isn't enough about them in any of the standards to criticize. Only 15 states even mention common denominators. The problem is that without a solid foundation in fractions, students have little hope of succeeding in college-level mathematics.

Although it may seem that I'm ignoring or skipping over high school mathematics, it's rather that I'm focusing on arithmetic as the needed foundation—and that foundation is taught in elementary school. You see, I'm not a "back-to-basics" person. Rather, I'm a "build-the-foundation" person.

The Common Core Standards: How Do They Stack Up?

For the most part, state standards are a thing of the past because most states have adopted the Common Core State Standards. So how will that change things?

The Common Core standards put a majority of state standards to shame. For one thing, they set arithmetic as a priority. Students have to memorize some number facts and learn the standard algorithms. For example, we see that by the end of grade 2, students must know from memory all the sums of any two one-digit numbers. However, those very same standards don't stipulate committing the corresponding subtractions to memory. Teachers will have to take care of that on their own. The same thing happens in grade 3 for the multiplication tables—there's no division. What were the standards creators thinking?

In grade 3, the focus on addition and subtraction continues. There we have, "Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction." That phrase "strategies and algorithms" is potentially a problem, though; it sounds like an attempt to undermine the standard algorithms.

Finally, in grade 4, we have the proper capstone standard for adding and subtracting whole numbers: "Fluently add and subtract multidigit whole numbers using the standard algorithm." However, doing this in 4th grade *after* students have nailed down fluency with adding and subtracting—using who-knows-what technique—is problematic

in many ways. I doubt if it's easy to get kids to change how they do things once they're doing them fluently. This whole discussion gets repeated for multiplication.

Where Common Core really shines is with fractions. In grade 3, fractions are actually defined as numbers! Fractions are usually introduced as parts of cookies or pizzas, but ultimately fractions must be numbers. This transition from pizzas to numbers is often missing. A clear definition of fractions as numbers allows the usual four arithmetic operations on fractions to make sense in grades 5 and 6. The rigor of the Common Core standards should help teachers teach fractions. With luck, it will force textbook writers to learn what fractions really are and develop them rigorously in math textbooks in a way that makes mathematical sense to both students and teachers.

With Common Core Standards, we have embarked on a major change of expectations for education in the United States. How will it turn out? Only time will tell.

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