Washington State High School Mathematics Standards

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Office of the Superintendent

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Overview

The *Washington State K–12 Mathematics Standards* outline the mathematics learning expectations for all students in Washington. These standards describe the mathematics content, procedures, applications, and processes that students are expected to learn. The topics and mathematical strands represented across grades K–12 constitute a mathematically complete program that includes the study of numbers, operations, geometry, measurement, algebra, data analysis, and important mathematical processes.

Organization of the standards

The *Washington State K–12 Mathematics Standards* are organized by grade level for grades K–8 and by course for grades 9–12, with each grade/course consisting of three elements: *Core Content*, *Additional Key Content*, and *Core Processes*. Each of these elements contains *Performance Expectations* and *Explanatory Comments and Examples*.

Core Content areas describe the major mathematical focuses of each grade level or course. A limited number of priorities for each grade level in grades K–8 and for each high school course are identified, so teachers know which topics call for the most time and emphasis. Each priority area includes a descriptive paragraph that highlights the mathematics addressed and its role in a student's overall mathematics learning.

Additional Key Content contains important expectations that do not warrant the same amount of instructional time as the Core Content areas. These are expectations that might extend a previously learned skill, plant a seed for future development, or address a focused topic, such as scientific notation. Although they need less classroom time, these expectations are important, are expected to be taught, and may be assessed as part of Washington State's assessment system. The content in this section allows students to build a coherent knowledge of mathematics from year to year.

Core Processes include expectations that address reasoning, problem solving, and communication. While these processes are incorporated throughout other content expectations, they are presented in this section to clearly describe the breadth and scope of what is expected in each grade or course. In Core Processes, at least two rich problems that cut across Core or Key Content areas are included as examples for each grade or course. These problems illustrate the types and breadth of problems that could be used in the classroom.

Performance Expectations, in keeping with the accepted definition of standards, describe what students should know and be able to do at each grade level. These statements are the core of the document. They are designed to provide clear guidance to teachers about the mathematics that is to be taught and learned.

Explanatory Comments and Examples accompany most of the expectations. These are not technically performance expectations. However, taken together with the Performance Expectations, they provide a full context and clear understanding of the expectation.

The comments expand upon the meaning of the expectations. Explanatory text might clarify the parameters regarding the type or size of numbers, provide more information about student expectations regarding mathematical understanding, or give expanded detail to mathematical definitions, laws, principles, and forms included in the expectation.

The example problems include those that are typical of the problems students should do, those that illustrate various types of problems associated with a particular performance expectation, and those that illustrate the expected limits of difficulty for problems related to a performance expectation. Teachers are not expected to teach these particular examples or to limit what they teach to these examples. Teachers and quality instructional materials will incorporate many different types of examples that support the teaching of the content described in any expectation.

In some instances, comments related to pedagogy are included in the standards as familiar illustrations to the teacher. Teachers are not expected to use these particular teaching methods or to limit the methods they use to the methods included in the document. These, too, are illustrative, showing one way an expectation might be taught.

Although, technically, the performance expectations set the requirements for Washington students, people will consider the entire document as the Washington mathematics standards. Thus, the term *standards*, as used here, refers to the complete set of Performance Expectations, Explanatory Comments and Examples, Core Content, Additional Key Content, and Core Processes. Making sense of the standards from any grade level or course calls for understanding the interplay of Core Content, Additional Key Content, and Core Processes for that grade or course.

What standards are not

Performance expectations do not describe how the mathematics will be taught. Decisions about instructional methods and materials are left to professional teachers who are knowledgeable about the mathematics being taught and about the needs of their students.

The standards are not comprehensive. They do not describe everything that could be taught in a classroom. Teachers may choose to go beyond what is included in this document to provide related or supporting content. They should teach beyond the standards to those students ready for additional challenges. Standards related to number skills, in particular, should be viewed as a floor—minimum expectations—and not a ceiling. A student who can order and compare numbers to 120 should be given every opportunity to apply these concepts to larger numbers.

The standards are not test specifications. Excessive detail, such as the size of numbers that can be tested and the conditions for assessment, clouds the clarity and usability of a standards document, generally, and a performance expectation, specifically. For example, it is sufficient to say "Identify, describe, and classify triangles by angle measure and number of congruent sides," without specifying that acute, right, and obtuse are types of triangles classified by their angle size and that scalene, isosceles, and equilateral are types of triangles classified by their side length. Sometimes this type of information is included in the comments section, but generally this level of detail is left to other documents.

What about strands?

Many states' standards are organized around mathematical content strands—generally some combination of numbers, operations, geometry, measurement, algebra, and data/statistics. However, the *Washington State K–12 Mathematics Standards* are organized according to the priorities described as Core Content rather than being organized in strands. Nevertheless, it is still useful to know what content strands are addressed in particular Core Content and Additional Key Content areas. Thus, mathematics content strands are identified in parentheses at the beginning of each Core Content or Additional Key Content area. Five content strands have been identified for this purpose: Numbers, Operations, Geometry/Measurement, Algebra, and Data/Statistics/Probability. For each of these strands, a separate K–12 strand document allows teachers and other readers to track the development of knowledge and skills across grades and courses. An additional strand document on the Core Processes tracks the development of reasoning, problem solving, and communication across grades K–12.

A well-balanced mathematics program for all students

An effective mathematics program balances three important components of mathematics—conceptual understanding (making sense of mathematics), procedural proficiency (skills, facts, and procedures), and problem solving and mathematical processes (using mathematics to reason, think, and apply mathematical knowledge). These standards make clear the importance of all three of these components, purposefully interwoven to support students' development as increasingly sophisticated mathematical thinkers. The standards are written to support the development of students so that they know and understand mathematics.

Conceptual understanding (making sense of mathematics)

Students who understand a concept are able to identify examples as well as non-examples, describe the concept (for example, with words, symbols, drawings, tables, or models), provide a definition of the concept, and use the concept in different ways. Conceptual understanding is woven throughout these standards. Expectations with verbs like *demonstrate*, *describe*, *represent*, *connect*, and *justify*, for example, ask students to show their understanding. Furthermore, expectations addressing both procedures and applications often ask students to connect their conceptual understanding to the procedures being learned or problems being solved.

Procedural proficiency (skills, facts, and procedures)

Learning basic facts is important for developing mathematical understanding. In these standards, clear expectations address students' knowledge of basic facts. The use of the term *basic facts* typically encompasses addition and multiplication facts up to and including 10 + 10 and 10 x 10 and their related subtraction and division facts. In these standards, students are expected to "quickly recall" basic facts. "Quickly recall" means that the student has ready and effective access to facts without having to go through a development process or strategy, such as counting up or drawing a picture, every time he or she needs to know a fact. Simply put, students need to know their basic facts.

Building on a sound conceptual understanding of addition, subtraction, multiplication, and division, Washington's standards include a specific discussion of students' need to understand and use the standard algorithms generally seen in the United States to add, subtract, multiply, and divide whole numbers. There are other possible algorithms students might also use to perform these operations and some teachers may find value in students learning multiple algorithms to enhance understanding.

Algorithms are step-by-step mathematical procedures that, if followed correctly, always produce a correct solution or answer. Generalized procedures are used throughout mathematics, such as in drawing geometric constructions or going through the steps involved in solving an algebraic equation. Students should come to understand that mathematical procedures are a useful and important part of mathematics.

The term *fluency* is used in these standards to describe the expected level and depth of a student's knowledge of a computational procedure. For the purposes of these standards, a student is considered fluent when the procedure can be performed immediately and accurately. Also, when fluent, the student knows when it is appropriate to use a particular procedure in a problem or situation. A student who is fluent in a procedure has a tool that can be applied reflexively and doesn't distract from the task of solving the problem at hand. The procedure is stored in long-term memory, leaving working memory available to focus on the problem.

Problem solving and mathematical processes (reasoning and thinking to apply mathematical content)

Mathematical processes, including reasoning, problem solving, and communication, are essential in a well-balanced mathematics program. Students must be able to reason, solve problems, and communicate their understanding in effective ways. While it is impossible to completely separate processes and content, the standards' explicit description of processes at each grade level calls attention to their importance within a well-balanced mathematics program. Some common language is used to describe the Core Processes across the grades and within grade bands (K–2, 3–5, 6–8, and 9–12). The problems students will address, as well as the language and symbolism they will use to communicate their mathematical understanding, become more sophisticated from grade to grade. These shifts across the grades reflect the increasing complexity of content and the increasing rigor as students deal with more challenging problems, much in the same way that reading skills develop from grade to grade with increasingly complex reading material.

Technology

The role of technology in learning mathematics is a complex issue, because of the ever-changing capabilities of technological tools, differing beliefs in the contributions of technology to a student's education, and equitable student access to tools. However, one principle remains constant: The focus of mathematics instruction should always be on the mathematics to be learned and on helping students learn that mathematics.

Technology ***should be used*** *when it supports the mathematics to be learned, and technology* ***should not be used*** *when it might interfere with learning.*

Calculators and other technological tools, such as computer algebra systems, dynamic geometry software, applets, spreadsheets, and interactive presentation devices are an important part of today's classroom. But the use of technology cannot replace conceptual understanding, computational fluency, or problem-solving skills.

Washington's standards make clear that some performance expectations are to be done without the aid of technology. Elementary students are expected to know facts and basic computational procedures without using a calculator. At the secondary level, students should compute with polynomials, solve equations, sketch simple graphs, and perform some constructions without the use of technology. Students should continue to use previously learned facts and skills in subsequent grade levels to maintain their fluency without the assistance of a calculator.

At the elementary level, calculators are less useful than they will be in later grades. The core of elementary school—number sense and computational fluency—does not require a calculator. However, this is not to say that students couldn't use calculators to investigate mathematical situations and to solve problems involving complicated numbers, lots of numbers, or data sets.

As middle school students deal with increasingly complex statistical data and represent proportional relationships with graphs and tables, a calculator or technological tool with these functions can be useful for representing relationships in multiple ways. At the high school level, graphing calculators become valuable tools as all students tackle the challenges of algebra and geometry to prepare for a range of postsecondary options in a technological world. Graphing calculators and spreadsheets allow students to explore and solve problems with classes of functions in ways that were previously impossible.

While the majority of performance expectations describe skills and knowledge that a student could demonstrate without technology, learning when it is helpful to use these tools and when it is cumbersome is part of becoming mathematically literate. When students become dependent upon technology to solve basic math problems, the focus of mathematics instruction to help students learn mathematics has failed.

Connecting to the Washington Essential Academic Learning Requirements (EALRs) and Grade Level Expectations (GLEs)

The new *Washington State K–12 Mathematics Standards* continue Washington's longstanding commitment to teaching mathematics content and mathematical thinking. The new standards replace the former Essential Academic Learning Requirements (EALRs) and Grade Level Expectations (GLEs). The former mathematics EALRs, listed below, represent threads in the mathematical content, reasoning, problem solving, and communication that are reflected in these new standards.

EALR 1: The student understands and applies the concepts and procedures of mathematics.

EALR 2: The student uses mathematics to define and solve problems.

EALR 3: The student uses mathematical reasoning.

EALR 4: The student communicates knowledge and understanding in both everyday and mathematical language.

EALR 5: The student understands how mathematical ideas connect within mathematics, to other subjects.

System-wide standards implementation activities

These mathematics standards represent an important step in ramping up mathematics teaching and learning in the state. The standards provide a critical foundation, but are only the first step. Their success will depend on the implementation efforts that match many of the activities outlined in Washington's Joint Mathematics Action Plan. This includes attention to:

- Aligning the Washington Assessment for Student Learning to these standards;
- Identifying mathematics curriculum and instructional support materials;
- Providing systematic professional development so that instruction aligns with the standards;

- Developing online availability of the standards in various forms and formats, with additional example problems, classroom activities, and possible lessons embedded.

As with any comprehensive initiative, fully implementing these standards will not occur overnight. This implementation process will take time, as teachers become more familiar with the standards and as students enter each grade having learned more of the standards from previous grades. There is always a tension of balancing the need to raise the bar with the reality of how much change is possible, and how quickly this change can be implemented in real schools with real teachers and real students.

Change is hard. These standards expect more of students and more of their teachers. Still, if Washington's students are to be prepared to be competitive and to reach their highest potential, implementing these standards will pay off for years to come.

Algebra 1

A1.1. Core Content: *Solving Problems* *(Algebra)*

Students learn to solve many new types of problems in Algebra 1, and this first core content area highlights the types of problems students will be able to solve after they master the concepts and skills in this course. Students are introduced to several types of functions, including exponential and functions defined piecewise, and they spend considerable time with linear and quadratic functions. Each type of function included in Algebra I provides students a tool to solve yet another class of problems. They learn that specific functions model situations described in word problems, and so functions are used to solve various types of problems. The ability to determine functions and write equations that represent problems is an important mathematical skill in itself, Many problems that initially appear to be very different from each other can actually be represented by identical equations. Students encounter this important and unifying principle of algebra—that the same algebraic techniques can be applied to a wide variety of different situations.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A1.1.A Select and justify functions and equations to model and solve problems.	Students can analyze the rate of change of a function represented with a table or graph to determine if the function is linear. Students also analyze common ratios to determine if the function is exponential. After selecting a function to model a situation, students describe appropriate domain restrictions. They use the function to solve the problem and interpret the solution in the context of the original situation. Examples: • A cup is 6 cm tall, including a 1.1 cm lip. Find a function that represents the height of a stack of cups in terms of the number of cups in the stack. Find a function that represents the number of cups in a stack of a given height. • For the month of July, Michelle will be dog-sitting for her very wealthy, but eccentric, neighbor, Mrs. Buffett. Mrs. Buffett offers Michelle two different salary plans: — Plan 1: \$100 per day for the 31 days of the month. — Plan 2: \$1 for July 1, \$2 for July 2, \$4 for July 3, and so on, with the daily rate doubling each day. a. Write functions that model the amount of money Michelle will earn each day on Plan 1 and Plan 2. Justify the functions you wrote. b. State an appropriate domain for each of the models based on the context. c. Which plan should Michelle choose to maximize her earnings? Justify your recommendation mathematically.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
	d. Extension: Write an algebraic function for the cumulative pay for each plan based on the number of days worked.
A1.1.B Solve problems that can be represented by linear functions, equations, and inequalities.	It is mathematically important to represent a word problem as an equation. Students must analyze the situation and find a way to represent it mathematically. After solving the equation, students think about the solution in terms of the original problem. Examples: • The assistant pizza maker makes 6 pizzas an hour. The master pizza maker makes 10 pizzas an hour but starts baking two hours later than his assistant. Together, they must make 92 pizzas. How many hours from when the assistant starts baking will it take? What is a general equation, in function form, that could be used to determine the number of pizzas that can be made in two or more hours? • A swimming pool holds 375,000 liters of water. Two large hoses are used to fill the pool. The first hose fills at the rate of 1,500 liters per hour and the second hose fills at the rate of 2,000 liters per hour. How many hours does it take to fill the pool completely?
A1.1.C Solve problems that can be represented by a system of two linear equations or inequalities.	Examples: • An airplane flies from Baltimore to Seattle (assume a distance of 2,400 miles) in 7 hours, but the return flight takes only $4\frac{1}{2}$ hours. The air speed of the plane is the same in both directions. How many miles per hour does the plane fly with respect to the wind? What is the wind speed in miles per hour? • A coffee shop employee has one cup of 85% milk (the rest is chocolate) and another cup of 60% milk (the rest is chocolate). He wants to make one cup of 70% milk How much of the 85% milk and 60% milk should he mix together to make the 70% milk? • Two plumbing companies charge different rates for their service. Clyde's Plumbing Company charges a $75-per-visit fee that includes one hour of labor plus $45 dollars per hour after the first hour. We-Unclog-It Plumbers charges a $100-per-visit fee that includes one hour of labor plus $40 per hour after the first hour. For how many hours of plumbing work would Clyde's be less expensive than We-Unclog-It? Note: Although this context is discrete, students can model it with continuous linear functions.

Performance Expectation *Students are expected to:*	**Explanatory Comments and Examples**

A1.1.D Solve problems that can be represented by quadratic functions and equations.

Examples:

- Find the solutions to the simultaneous equations $y = x + 2$ and $y = x^2$.
- If you throw a ball straight up (with initial height of 4 feet) at 10 feet per second, how long will it take to fall back to the starting point? The function $h(t) = -16t^2 + v_0t + h_0$ describes the height, h in feet, of an object after t seconds, with initial velocity v_0 and initial height h_0.
- Joe owns a small plot of land 20 feet by 30 feet. He wants to double the area by increasing both the length and the width, keeping the dimensions in the same proportion as the original. What will be the new length and width?
- What two consecutive numbers, when multiplied together, give the first number plus 16? Write the equation that represents the situation.

A1.1.E Solve problems that can be represented by exponential functions and equations.

Students approximate solutions with graphs or tables, check solutions numerically, and when possible, solve problems exactly.

Examples:

- E. coli bacteria reproduce by a simple process called binary fission—each cell increases in size and divides into two cells. In the laboratory, E. coli bacteria divide approximately every 15 minutes. A new E. coli culture is started with 1 cell.
  a. Find a function that models the E. coli population size at the end of each 15-minute interval. Justify the function you found.
  b. State an appropriate domain for the model based on the context.
  c. After what 15-minute interval will you have at least 500 bacteria?
- Estimate the solution to $2^x = 16,384$

Algebra 1

A1.2. Core Content: *Numbers, expressions, and operations* *(Numbers, Operations, Algebra)*

Students see the number system extended to the real numbers represented by the number line. They work with integer exponents, scientific notation, and radicals, and use variables and expressions to solve problems from purely mathematical as well as applied contexts. They build on their understanding of computation using arithmetic operations and properties and expand this understanding to include the symbolic language of algebra. Students demonstrate this ability to write and manipulate a wide variety of algebraic expressions throughout high school mathematics as they apply algebraic procedures to solve problems.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A1.2.A Know the relationship between real numbers and the number line, and compare and order real numbers with and without the number line.	Although a formal definition of real numbers is beyond the scope of Algebra 1, students learn that every point on the number line represents a real number, either rational or irrational, and that every real number has its unique point on the number line. They locate, compare, and order real numbers on the number line.

Real numbers include those written in scientific notation or expressed as fractions, decimals, exponentials, or roots.

Examples:

- Without using a calculator, order the following on the number line:

  $\sqrt{82}$, 3π, 8.9, 9, $\frac{37}{4}$, 9.3×10^0

- A star's color gives an indication of its temperature and age. The chart shows four types of stars and the lowest temperature of each type.

Type	Lowest Temperature (in °F)	Color
A	1.35×10^4	Blue-White
B	2.08×10^4	Blue
G	9.0×10^3	Yellow
P	4.5×10^4	Blue

  List the temperatures in order from lowest to highest.

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
A1.2.B Recognize the multiple uses of variables, determine all possible values of variables that satisfy prescribed conditions, and evaluate algebraic expressions that involve variables.	Students learn to use letters as variables and in other ways that increase in sophistication throughout high school. For example, students learn that letters can be used: • To represent fixed and temporarily unknown values in equations, such as $3x + 2 = 5$; • To express identities, such as $x + x = 2x$ for all x; • As attributes in formulas, such as $A = lw$; • As constants such as a, b, and c in the equation $y = ax^2 + bx + c$; • As parameters in equations, such as the m and b for the family of functions defined by $y = mx + b$; • To represent varying quantities, such as x in $f(x) = 5x$; • To represent functions, such as f in $f(x) = 5x$; and • To represent specific numbers, such as π. Expressions include those involving polynomials, radicals, absolute values, and integer exponents. Examples: • For what values of a and n, where n is an integer greater than 0, is a^n always negative? • For what values of a is $\frac{1}{a}$ an integer? • For what values of a is $\sqrt{5-a}$ defined? • For what values of a is $-a$ always positive?
A1.2.C Interpret and use integer exponents and square and cube roots, and apply the laws and properties of exponents to simplify and evaluate exponential expressions.	Examples: • $2^{-3} = \frac{1}{2^3}$ • $\frac{2^{-2}3^{2}5}{2^{2}3^{-3}5^{2}} = \frac{3^5}{2^4 5}$ • $\frac{a^{-2}b^{2}c}{a^{2}b^{-3}c^{2}} = \frac{b^5}{a^4 c}$ • $\sqrt{8} = \sqrt{2 \bullet 2 \bullet 2} = 2\sqrt{2}$ • $\sqrt[3]{a \bullet b} = \sqrt[3]{a} \bullet \sqrt[3]{b}$

Performance Expectation *Students are expected to:*	*Explanatory Comments and Examples*

A1.2.D Determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection.

Decimal approximations of numbers are sometimes used in applications such as carpentry or engineering, while at other times, these applications may require exact values. Students should understand the difference and know that the appropriate approximation depends upon the necessary degree of precision needed in given situations.

For example, 1.414 is an approximation and not an exact solution to the equation $x^2 – 2 = 0$, but $\sqrt{2}$ is an exact solution to this equation.

Example:

- Using a common engineering formula, an engineering student represented the maximum safe load of a bridge to be $1000(99 – 70\sqrt{2})$ tons. He used 1.41 as the approximation for $\sqrt{2}$ in his calculations. When the bridge was built and tested in a computer simulation to verify its maximum weight-bearing load, it collapsed! The student had estimated the bridge would hold ten times the weight that was applied to it when it collapsed.
  - — Calculate the weight that the student thought the bridge could bear using 1.41 as the estimate for $\sqrt{2}$.
  - — Calculate other weight values using estimates of $\sqrt{2}$ that have more decimal places. What might be a reasonable degree of precision required to know how much weight the bridge can handle safely? Justify your answer.

A1.2.E Use algebraic properties to factor and combine like terms in polynomials.

Algebraic properties include the commutative, associative, and distributive properties.

Factoring includes:

- Factoring a monomial from a polynomial, such as $4x^2 + 6x = 2x(2x + 3)$;
- Factoring the difference of two squares, such as $36x^2 – 25y^2 = (6x + 5y)(6x – 5y)$ and $x^4 – y^4 = (x + y)(x – y)(x^2 + y^2)$;
- Factoring perfect square trinomials, such as $x^2 + 6xy + 9y^2 = (x + 3y)^2$;
- Factoring quadratic trinomials such as $x^2 + 5x + 4 = (x + 4)(x + 1)$; and

Performance Expectation	*Explanatory Comments and Examples*
Students are expected to:	
	• Factoring trinomials that can be expressed as the product of a constant and a trinomial, such as $0.5x^2 - 2.5x - 7 = 0.5(x + 2)(x - 7)$.
A1.2.F Add, subtract, multiply, and divide polynomials.	Write algebraic expressions in equivalent forms using algebraic properties to perform the four arithmetic operations with polynomials. Students should recognize that expressions are essentially sums, products, differences, or quotients. For example, the sum $2x^2 + 4x$ can be written as a product, $2x(x + 2)$. Examples: • $(3x^2 - 4x + 5) + (-x^2 + x - 4) + (2x^2 + 2x + 1)$ • $(2x^2 - 4) - (x^2 + 3x - 3)$ • $\frac{2x^2}{9} \bullet \frac{6}{2x^4}$ • $\frac{x^2 - 2x - 3}{x + 1}$

Algebra 1

A1.3. Core Content: *Characteristics and behaviors of functions* *(Algebra)*

Students formalize and deepen their understanding of functions, the defining characteristics and uses of functions, and the mathematical language used to describe functions. They learn that functions are often specified by an equation of the form $y = f(x)$, where any allowable x-value yields a unique y-value. While Algebra 1 has a particular focus on linear and quadratic equations and systems of equations, students also learn about exponential functions and those that can be defined piecewise, particularly step functions and functions that contain the absolute value of an expression. Students learn about the representations and basic transformations of these functions and the practical and mathematical limitations that must be considered when working with functions and when using functions to model situations.

Performance Expectation

Students are expected to:

Explanatory Comments and Examples

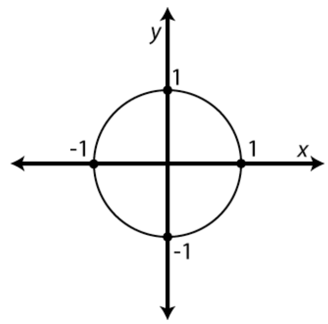
A1.3.A Determine whether a relationship is a function and identify the domain, range, roots, and independent and dependent variables.

Functions studied in Algebra 1 include linear, quadratic, exponential, and those defined piecewise (including step functions and those that contain the absolute value of an expression).

Given a problem situation, students should describe further restrictions on the domain of a function that are appropriate for the problem context.

Examples:

- Which of the following are functions? Explain why or why not.
  - — The age in years of each student in your math class and each student's shoe size.
  - — The number of degrees a person rotates a spigot and the volume of water that comes out of the spigot.
- A function $f(n) = 60n$ is used to model the distance in miles traveled by a car traveling 60 miles per hour in n hours. Identify the domain and range of this function. What restrictions on the domain of this function should be considered for the model to correctly reflect the situation?
- What is the domain of $f(x) = \sqrt{5 - x}$?
- Which of the following equations, inequalities, or graphs determine y as a function of x?
  - — $y = 2$
  - — $x = 3$
  - — $y = |x|$

Performance Expectation	*Explanatory Comments and Examples*
Students are expected to:	
	— $y = \begin{cases} x+3, & x \le 1 \\ x-2, & x > 1 \end{cases}$ — $x^2 + y^2 = 1$ 
A1.3.B Represent a function with a symbolic expression, as a graph, in a table, and using words, and make connections among these representations.	This expectation applies each time a new class (family) of functions is encountered. In Algebra 1, students should be introduced to a variety of additional functions that include expressions such as x^3, $\sqrt{x}$, $\frac{1}{x}$, and absolute values. They will study these functions in depth in subsequent courses. Students should know that f(x) = $\frac{a}{x}$ represents an inverse variation. Students begin to describe the graph of a function from its symbolic expression, and use key characteristics of the graph of a function to infer properties of the related symbolic expression. Translating among these various representations of functions is an important way to demonstrate conceptual understanding of functions. Students learn that each representation has particular advantages and limitations. For example, a graph shows the shape of a function, but not exact values. They also learn that a table of values may not uniquely determine a single function without some specification of the nature of that function (e.g., it is quadratic).
A1.3.C Evaluate $f(x)$ at a (i.e., $f(a)$) and solve for x in the equation $f(x) = b$.	Functions may be described and evaluated with symbolic expressions, tables, graphs, or verbal descriptions. Students should distinguish between solving for $f(x)$ and evaluating a function at x.

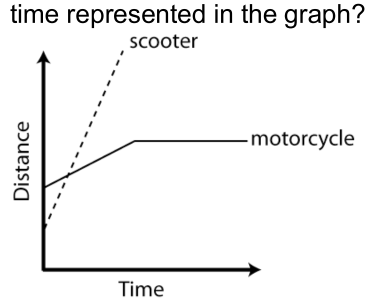
Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
	Example: • Roses-R-Red sells its roses for $0.75 per stem and charges a $20 delivery fee per order. — What is the cost of having 10 roses delivered? — How many roses can you have delivered for $65?

Algebra 1

***A1.4. Core Content:** Linear functions, equations, and inequalities* *(Algebra)*

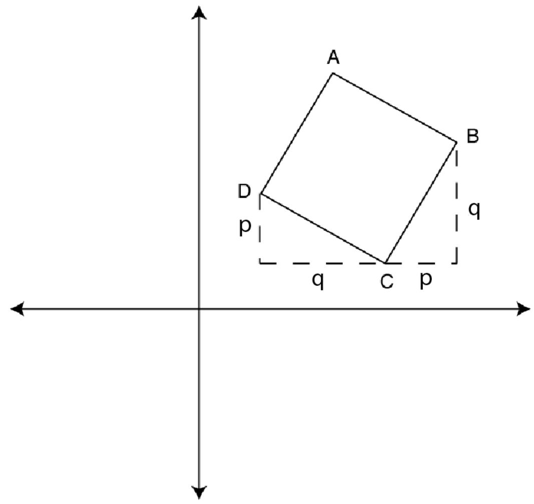
Students understand that linear functions can be used to model situations involving a constant rate of change. They build on the work done in middle school to solve sets of linear equations and inequalities in two variables, learning to interpret the intersection of the lines as the solution. While the focus is on solving equations, students also learn graphical and numerical methods for approximating solutions to equations. They use linear functions to analyze relationships, represent and model problems, and answer questions. These algebraic skills are applied in other Core Content areas across high school courses.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A1.4.A Write and solve linear equations and inequalities in one variable.	This expectation includes the use of absolute values in the equations and inequalities. Examples: • Write an absolute value equation or inequality for — all the numbers 2 units from 7, and — all the numbers that are more than *b* units from *a*. • Solve $\lvert x – 6\rvert \leq 4$ and locate the solution on the number line. • Write an equation or inequality that has — no real solutions; — infinite numbers of real solutions; and — exactly one real solution. • Solve for x in $2(x – 3) + 4x = 15 + 2x$. • Solve $8.5 < 3x + 2 \leq 9.7$ and locate the solution on the number line.
A1.4.B Write and graph an equation for a line given the slope and the *y*-intercept, the slope and a point on the line, or two points on the line, and translate between forms of linear equations.	Linear equations may be written in slope-intercept, point-slope, and standard form. Examples: • Find an equation for a line with *y*-intercept equal to 2 and slope equal to 3. • Find an equation for a line with a slope of 2 that goes through the point (1, 1). • Find an equation for a line that goes through the points (-3, 5) and (6, -2).

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
	• For each of the following, use only the equation (without sketching the graph) to describe the graph. — $y = 2x + 3$ — $y - 7 = 2(x - 2)$ • Write the equation $3x + 2y = 5$ in slope intercept form. • Write the equation $y - 1 = 2(x - 2)$ in standard form.
A1.4.C Identify and interpret the slope and intercepts of a linear function, including equations for parallel and perpendicular lines.	Examples: • The graph shows the relationship between time and distance from a gas station for a motorcycle and a scooter. What can be said about the relative speed of the motorcycle and scooter that matches the information in the graph? What can be said about the intersection of the graphs of the scooter and the motorcycle? Is it possible to tell which vehicle is further from the gas station at the initial starting point represented in the graph? At the end of the time represented in the graph? Why or why not?  • A 1,500-gallon tank contains 200 gallons of water. Water begins to run into the tank at the rate of 75 gallons per hour. When will the tank be full? Find a linear function that models this situation, draw a graph, and create a table of data points. Once you have answered the question and completed the tasks, explain your reasoning. Interpret the slope and y-intercept of the function in the context of the situation.

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples

- Given that the figure below is a square, find the slope of the perpendicular sides AB and BC. Describe the relationship between the two slopes.



A1.4.D Write and solve systems of two linear equations and inequalities in two variables.

Students solve both symbolic and word problems, understanding that the solution to a problem is given by the coordinates of the intersection of the two lines when the lines are graphed in the same coordinate plane.

Examples:

- Solve the following simultaneous linear equations algebraically:
  - $-2x + y = 2$
  - $x + y = -1$
- Graph the above two linear equations on the same coordinate plane and use the graph to verify the algebraic solution.
- An academic team is going to a state mathematics competition. There are 30 people going on the trip. There are 5 people who can drive and 2 types of vehicles, vans and cars. A van seats 8 people, and a car seats 4 people, including drivers. How many vans and cars does the team need for the trip? Explain your reasoning.

  Let v = number of vans and c = number of cars.

  $v + c \leq 5$

  $8v + 4c \geq 30$

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
A1.4.E Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships they represent.	In the case of a linear function $y = f(x)$, expressed in slope-intercept form ($y = mx + b$), m and b are parameters. Students should know that $f(x) = kx$ represents a direct variation (proportional relationship). Examples: • Graph a function of the form $f(x) = kx$, describe the effect that changes on k have on the graph and on $f(x)$, and answer questions that arise in proportional situations. • A gas station's 10,000-gallon underground storage tank contains 1,000 gallons of gasoline. Tanker trucks pump gasoline into the tank at a rate of 400 gallons per minute. How long will it take to fill the tank? Find a function that represents this situation and then graph the function. If the flow rate increases from 400 to 500 gallons per minute, how will the graph of the function change? If the initial amount of gasoline in the tank changes from 1,000 to 2,000 gallons, how will the graph of the function change? • Compare and contrast the functions $y = 3\|x\|$ and $y = -\frac{1}{3}\|x\|$.

Algebra 1

A1.5. Core Content: *Quadratic functions and equations* *(Algebra)*

Students study quadratic functions and their graphs, and solve quadratic equations with real roots in Algebra 1. They use quadratic functions to represent and model problems and answer questions in situations that are modeled by these functions. Students solve quadratic equations by factoring and computing with polynomials. The important mathematical technique of completing the square is developed enough so that the quadratic formula can be derived.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A1.5.A Represent a quadratic function with a symbolic expression, as a graph, in a table, and with a description, and make connections among the representations.	Example: • Kendre and Tyra built a tennis ball cannon that launches tennis balls straight up in the air at an initial velocity of 50 feet per second. The mouth of the cannon is 2 feet off the ground. The function $h(t) = -16t^2 + 50t + 2$ describes the height, h, in feet, of the ball t seconds after the launch. Make a table from the function. Then use the table to sketch a graph of the height of the tennis ball as a function of time into the launch. Give a verbal description of the graph. How high was the ball after 1 second? When does it reach this height again?
A1.5.B Sketch the graph of a quadratic function, describe the effects that changes in the parameters have on the graph, and interpret the *x*-intercepts as solutions to a quadratic equation.	Note that in Algebra 1, the parameter b in the term bx in the quadratic form $ax^2 + bx + c$ is not often used to provide useful information about the characteristics of the graph. Parameters considered most useful are: • a and c in $f(x) = ax^2 + c$ • a, h, and k in $f(x) = a(x - h)^2 + k$, and • a, r, and s in $f(x) = a(x - r)(x - s)$ Example: • A particular quadratic function can be expressed in the following two ways: $f(x) = -(x - 3)^2 + 1$ $f(x) = -(x - 2)(x - 4)$ — What information about the graph can be directly inferred from each of these forms? Explain your reasoning. — Sketch the graph of this function, showing the roots.

Performance Expectation *Students are expected to:*	*Explanatory Comments and Examples*

A1.5.C Solve quadratic equations that can be factored as $(ax + b)(cx + d)$ where a, b, c, and d are integers.

Students learn to efficiently solve quadratic equations by recognizing and using the simplest factoring methods, including recognizing special quadratics as squares and differences of squares.

Examples:

- $2x^2 + x - 3 = 0$; $(x - 1)(2x + 3) = 0$; $x = 1, -\frac{3}{2}$
- $4x^2 + 6x = 0$; $2x(2x + 3) = 0$; $x = 0, -\frac{3}{2}$
- $36x^2 - 25 = 0$; $(6x + 5)(6x - 5) = 0$; $x = \pm\frac{5}{6}$
- $x^2 + 6x + 9 = 0$; $(x + 3)^2 = 0$; $x = -3$

A1.5.D Solve quadratic equations that have real roots by completing the square and by using the quadratic formula.

Students solve those equations that are not easily factored by completing the square and by using the quadratic formula. Completing the square should also be used to derive the quadratic formula.

Students learn how to determine if there are two, one, or no real solutions.

Examples:

- Complete the square to solve $x^2 + 4x = 13$.

  $x^2 + 4x - 13 = 0$

  $x^2 + 4x + 4 = 17$

  $(x + 2)^2 = 17$

  $x + 2 = \pm\sqrt{17}$

  $x = -2 \pm\sqrt{17}$

  $x \approx 2.12, -6.12$

- Use the quadratic formula to solve $4x^2 - 2x = 5$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4 \bullet -5)}}{2(4)}$$

$$x = \frac{2 \pm \sqrt{84}}{8}$$

$$x = \frac{2 \pm 2\sqrt{21}}{8}$$

$$x = \frac{1 \pm \sqrt{21}}{4}$$

$x \approx 1.40, -0.90$

Algebra 1

***A1.6. Core Content:** Data and distributions* *(Data/Statistics/Probability)*

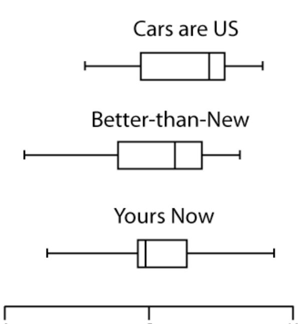
Students select mathematical models for data sets and use those models to represent, describe, and compare data sets. They analyze data to determine the relationship between two variables and make and defend appropriate predictions, conjectures, and generalizations. Students understand limitations of conclusions based on results of a study or experiment and recognize common misconceptions and misrepresentations in interpreting conclusions.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A1.6.A Use and evaluate the accuracy of summary statistics to describe and compare data sets.	A univariate set of data identifies data on a single variable, such as shoe size.

This expectation extends what students have learned in earlier grades to include evaluation and justification. They both compute and evaluate the appropriateness of measure of center and spread (range and interquartile range) and use these measures to accurately compare data sets. Students will draw appropriate conclusions through the use of statistical measures of center, frequency, and spread, combined with graphical displays.

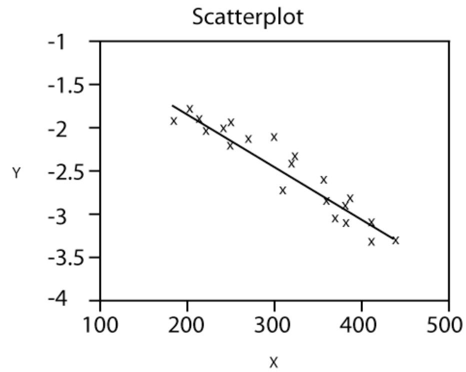
Examples:

- The local minor league baseball team has a salary dispute. Players claim they are being underpaid, but managers disagree.
  - — Bearing in mind that a few top players earn salaries that are quite high, would it be in the managers' best interest to use the mean or median when quoting the "average" salary of the team? Why?
  - — What would be in the players' best interest?
- Each box-and-whisker plot shows the prices of used cars (in thousands of dollars) advertised for sale at three different car dealers. If you want to go to the dealer whose prices seem least expensive, which dealer would you go to? Use statistics from the displays to justify your answer.



Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A1.6.B Make valid inferences and draw conclusions based on data.	Determine whether arguments based on data confuse association with causation. Evaluate the reasonableness of and make judgments about statistical claims, reports, studies, and conclusions. Example: • Mr. Shapiro found that the amount of time his students spent doing mathematics homework is positively correlated with test grades in his class. He concluded that doing homework makes students' test scores higher. Is this conclusion justified? Explain any flaws in Mr. Shapiro's reasoning.
A1.6.C Describe how linear transformations affect the center and spread of univariate data.	Examples: • A company decides to give every one of its employees a $5,000 raise. What happens to the mean and standard deviation of the salaries as a result? • A company decides to double each of its employee's salaries. What happens to the mean and standard deviation of the salaries as a result?
A1.6.D Find the equation of a linear function that best fits bivariate data that are linearly related, interpret the slope and *y*-intercept of the line, and use the equation to make predictions.	A bivariate set of data presents data on two variables, such as shoe size and height. In high school, the emphasis is on using a line of best fit to interpret data and on students making judgments about whether a bivariate data set can be modeled with a linear function. Students can use various methods, including technology, to obtain a line of best fit. Making predictions involves both interpolating and extrapolating from the original data set. Students need to be able to evaluate the quality of their predictions, recognizing that extrapolation is based on the assumption that the trend indicated continues beyond the unknown data.

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
A1.6.E Describe the correlation of data in scatterplots in terms of strong or weak and positive or negative.	Examples: • Which words—*strong* or *weak*, *positive* or *negative*—could be used to describe the correlation shown in the sample scatterplot below?



Algebra 1

A1.7. Additional Key Content *(Algebra)*

Students develop a basic understanding of arithmetic and geometric sequences and of exponential functions, including their graphs and other representations. They use exponential functions to analyze relationships, represent and model problems, and answer questions in situations that are modeled by these nonlinear functions. Students learn graphical and numerical methods for approximating solutions to exponential equations. Students interpret the meaning of problem solutions and explain limitations related to solutions.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A1.7.A Sketch the graph for an exponential function of the form $y = ab^n$ where n is an integer, describe the effects that changes in the parameters a and b have on the graph, and answer questions that arise in situations modeled by exponential functions.	Examples: • Sketch the graph of $y = 2^n$ by hand. • You have won a door prize and are given a choice between two options: $150 invested for 10 years at 4% compounded annually. $200 invested for 10 years at 3% compounded annually. — How much is each worth at the end of each year of the investment periods? — Are the two investments ever equal in value? Which will you choose?
A1.7.B Find and approximate solutions to exponential equations.	Students can approximate solutions using graphs or tables with and without technology.
A1.7.C Express arithmetic and geometric sequences in both explicit and recursive forms, translate between the two forms, explain how rate of change is represented in each form, and use the forms to find specific terms in the sequence.	Examples: • Write a recursive formula for the arithmetic sequence 5, 9, 13, 17, . . . What is the slope of the line that contains the points associated with these values and their position in the sequence? How is the slope of the line related to the sequence? • Given that $u(0) = 3$ and $u(n + 1) = u(n) + 7$ when n is a positive integer, a. find $u(5)$; b. find n so that $u(n) = 361$; and c. find a formula for $u(n)$. • Write a recursive formula for the geometric sequence 5, 10, 20, 40, . . . and determine the 100th term.

Performance Expectation *Students are expected to:*	*Explanatory Comments and Examples*
	• Given that $u(0) = 2$ and $u(n + 1) = 3u(n)$, a. find $u(4)$, and b. find a formula for $u(n)$.
A1.7.D Solve an equation involving several variables by expressing one variable in terms of the others.	Examples: • Solve $A = p + prt$ for p. • Solve $V = \pi r^2 h$ for h or for r.

Algebra 1

***A1.8. Core Processes:** Reasoning, problem solving, and communication*

Students formalize the development of reasoning in Algebra 1 as they use algebra and the properties of number systems to develop valid mathematical arguments, make and prove conjectures, and find counterexamples to refute false statements, using correct mathematical language, terms, and symbols in all situations. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students formalize a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

Performance Expectation

Students are expected to:

A1.8.A Analyze a problem situation and represent it mathematically.

A1.8.B Select and apply strategies to solve problems.

A1.8.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.

A1.8.D Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.

A1.8.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.

A1.8.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.

Explanatory Comments and Examples (applies to all expectations)

Examples:

- Three teams of students independently conducted experiments to relate the rebound height of a ball to the rebound number. The table gives the average of the teams' results.

  Construct a scatterplot of the data, and describe the function that relates the height of the ball to the rebound number. Predict the rebound height of the ball on the tenth rebound. Justify your answer.

Rebound Number	Rebound Height (cm)
0	200
1	155
2	116
3	88
4	66
5	50
6	44

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples (applies to all expectations)***
A1.8.G Synthesize information to draw conclusions, and evaluate the arguments and conclusions of others. A1.8.H Use inductive reasoning about algebra and the properties of numbers to make conjectures, and use deductive reasoning to prove or disprove conjectures.	• Prove $(a + b)^2 = a^2 + 2ab + b^2$. • A student writes $(x + 3)^2 = x^2 + 9$. Explain why this is incorrect. • Prove formally that the sum of two odd numbers is always even.

Geometry

***G.1. Core Content:** Logical arguments and proofs* *(Logic)*

Students formalize the reasoning skills they have developed in previous grades and solidify their understanding of what it means to prove a geometric statement mathematically. In Geometry, students encounter the concept of formal proof built on definitions, axioms, and theorems. They use inductive reasoning to test conjectures about geometric relationships and use deductive reasoning to prove or disprove their conclusions. Students defend their reasoning using precise mathematical language and symbols.

Performance Expectation *Students are expected to:*		***Explanatory Comments and Examples***
G.1.A	Distinguish between inductive and deductive reasoning.	Students generate and test conjectures inductively and then prove (or disprove) their conclusions deductively. Example: • A student first hypothesizes that the number of degrees in a polygon = 180 · (s – 2), where s represents the number of sides, and then proves this is true. When was the student using inductive reasoning? When was s/he using deductive reasoning? Justify your answers.
G.1.B	Use inductive reasoning to make conjectures, to test the plausibility of a geometric statement, and to help find a counterexample.	Examples: • Investigate the relationship among the medians of a triangle using paper folding. Make a conjecture about this relationship. • Using dynamic geometry software, decide if the following is a plausible conjecture: If segment AM is a median in triangle ABC, then ray AM bisects angle BAC.
G.1.C	Use deductive reasoning to prove that a valid geometric statement is true.	Valid proofs may be presented in paragraph, two-column, or flow-chart formats. Proof by contradiction is a form of deductive reasoning. Example: • Prove that the diagonals of a rhombus are perpendicular bisectors of each other.

Performance Expectation *Students are expected to:*		***Explanatory Comments and Examples***
G.1.D	Write the converse, inverse, and contrapositive of a valid proposition and determine their validity.	Examples: • If *m* and *n* are odd integers, then the sum of *m* and *n* is an even integer. State the converse and determine whether it is valid. • If a quadrilateral is a rectangle, the diagonals have the same length. State the contrapositive and determine whether it is valid.
G.1.E	Identify errors or gaps in a mathematical argument and develop counterexamples to refute invalid statements about geometric relationships.	Example: • Identify errors in reasoning in the following proof: Given $\angle ABC \cong \angle PRQ$, $\overline{AB} \cong \overline{PQ}$, and $\overline{BC} \cong \overline{QR}$, then $\Delta ABC \cong \Delta PQR$ by SAS.
G.1.F	Distinguish between definitions and undefined geometric terms and explain the role of definitions, undefined terms, postulates (axioms), and theorems.	Students sketch *points* and *lines* (undefined terms) and define and sketch representations of other common terms. They use definitions and postulates as they prove theorems throughout geometry. In their work with theorems, they identify the hypothesis and the conclusion and explain the role of each. Students describe the consequences of changing assumptions or using different definitions for subsequent theorems and logical arguments. Example: • There are two definitions of trapezoid that can be found in books or on the web. A trapezoid is either — a quadrilateral with exactly one pair of parallel sides or — a quadrilateral with at least one pair of parallel sides. Write some theorems that are true when applied to one definition but not the other, and explain your answer.

Geometry

G.2 Core Content: *Lines and Angles* *(Geometry/Measurement)*

Students study basic properties of parallel and perpendicular lines, their respective slopes, and the properties of the angles formed when parallel lines are intersected by a transversal. They prove related theorems and apply them to solve both mathematical and practical problems.

Performance Expectation *Students are expected to:*		***Explanatory Comments and Examples***
G.2.A	Know, prove, and apply theorems about parallel and perpendicular lines.	Students should be able to summarize and explain basic theorems. They are not expected to recite lists of theorems, but they should know the conclusion of a theorem when given its hypothesis. Examples: • Prove that a point on the perpendicular bisector of a line segment is equidistant from the ends of the line segment. • If each of two lines is perpendicular to a given line, what is the relationship between the two lines? How do you know?
G.2.B	Know, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal.	Example: • Prove that if two parallel lines are cut by a transversal, then alternate-interior angles are equal. • Take two parallel lines *l* and *m*, with (distinct) points A and B on *l* and C and D on *m*. If $\overleftrightarrow{AC}$ intersects $\overleftrightarrow{BD}$ at point E, prove that $\Delta ABE \approx \Delta CDE$.
G.2.C	Explain and perform basic compass and straightedge constructions related to parallel and perpendicular lines.	Constructions using circles and lines with dynamic geometry software (i.e., virtual compass and straightedge) are equivalent to paper and pencil constructions. Example: • Construct and mathematically justify the steps to: — Bisect a line segment. — Drop a perpendicular from a point to a line. — Construct a line through a point that is parallel to another line.
G.2.D	Describe the intersections of lines in the plane and in space, of lines and planes, and of planes in space.	Example: • Describe all the ways that three planes can intersect in space.

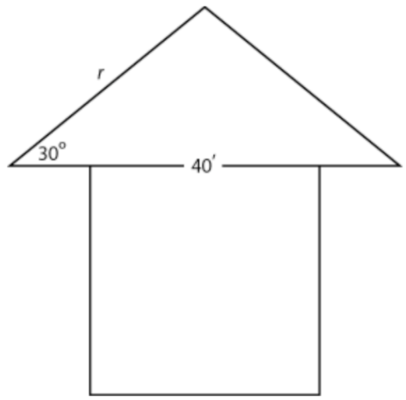
Geometry

G.3. Core Content: *Two- and three-dimensional figures* *(Geometry/Measurement)*

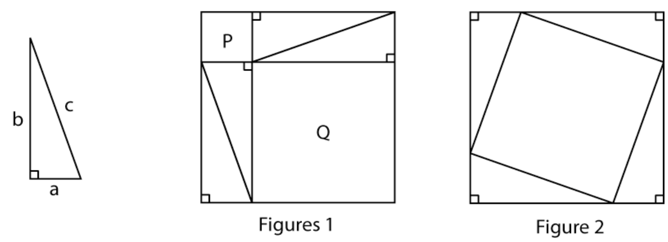
Students know and can prove theorems about two- and three-dimensional geometric figures, both formally and informally. They identify necessary and sufficient conditions for proving congruence, similarity, and properties of figures. Triangles are a primary focus, beginning with general properties of triangles, working with right triangles and special triangles, proving and applying the Pythagorean Theorem and its converse, and applying the basic trigonometric ratios of sine, cosine, and tangent. Students extend their learning to other polygons and the circle, and do some work with three-dimensional figures.

Performance Expectation *Students are expected to:*		***Explanatory Comments and Examples***
G.3.A	Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.	Examples: • Prove that the sum of the angles of a triangle is 180°. • Prove and explain theorems about the incenter, circumcenter, orthocenter, and centroid. • The rural towns of Atwood, Bridgeville, and Carnegie are building a communications tower to serve the needs of all three towns. They want to position the tower so that the distance from each town to the tower is equal. Where should they locate the tower? How far will it be from each town?
G.3.B	Determine and prove triangle congruence, triangle similarity, and other properties of triangles.	Students should identify necessary and sufficient conditions for congruence and similarity in triangles, and use these conditions in proofs. Examples: • Prove that congruent triangles are similar. • For a given ΔRST, prove that ΔXYZ, formed by joining the midpoints of the sides of ΔRST, is similar to ΔRST. • Show that not all SSA triangles are congruent.
G.3.C	Use the properties of special right triangles (30°–60°–90° and 45°–45°–90°) to solve problems.	Examples: • Determine the length of the altitude of an equilateral triangle whose side lengths measure 5 units. • If one leg of a right triangle has length 5 and the adjacent angle is 30°, what is the length of the other leg and the hypotenuse?

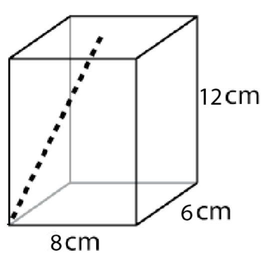
Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
	• If one leg of a 45°–45°–90° triangle has length 5, what is the length of the hypotenuse? • The pitch of a symmetrical roof on a house 40 feet wide is 30º. What is the length of the rafter, *r*, exactly and approximately?



G.3.D Know, prove, and apply the Pythagorean Theorem and its converse.	Examples: • Consider any right triangle with legs *a* and *b* and hypotenuse *c*. The right triangle is used to create Figures 1 and 2. Explain how these figures constitute a visual representation of a proof of the Pythagorean Theorem.



• A juice box is 6 cm by 8 cm by 12 cm. A straw is inserted into a hole in the center of the top of the box. The straw must stick out 2 cm so you can drink from it. If the straw must be long enough to touch each bottom corner of the box, what is the minimum length the straw must be? (Assume the diameter of the straw is 0 for the mathematical model.)



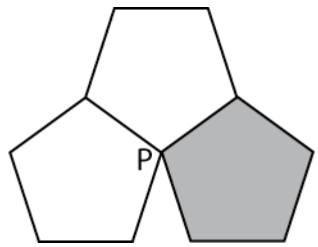
Performance Expectation *Students are expected to:*		Explanatory Comments and Examples
		• In ΔABC, with right angle at *C*, draw the altitude $\overline{CD}$ from C to $\overline{AB}$. Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem. • Apply the Pythagorean Theorem to derive the distance formula in the (*x*, *y*) plane.
G.3.E	Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent.	Examples: • A 12-foot ladder leans against a wall to form a 63° angle with the ground. How many feet above the ground is the point on the wall at which the ladder is resting? • Use the Pythagorean Theorem to establish that $\sin^2 ø + \cos^2 ø = 1$ for ø between 0° and 90°.
G.3.F	Know, prove, and apply basic theorems about parallelograms.	Properties may include those that address symmetry and properties of angles, diagonals, and angle sums. Students may use inductive and deductive reasoning and counterexamples. Examples: • Are opposite sides of a parallelogram always congruent? Why or why not? • Are opposite angles of a parallelogram always congruent? Why or why not? • Prove that the diagonals of a parallelogram bisect each other. • Explain why if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. • Prove that the diagonals of a rectangle are congruent. Is this true for any parallelogram? Justify your reasoning.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***

G.3.G Know, prove, and apply theorems about properties of quadrilaterals and other polygons.

Examples:

- What is the length of the apothem of a regular hexagon with side length 8? What is the area of the hexagon?
- If the shaded pentagon were removed, it could be replaced by a regular *n*-sided polygon that would exactly fill the remaining space. Find the number of sides, *n*, of a replacement polygon that makes the three polygons fit perfectly.



G.3.H Know, prove, and apply basic theorems relating circles to tangents, chords, radii, secants, and inscribed angles.

Examples:

- Given a line tangent to a circle, know and explain that the line is perpendicular to the radius drawn to the point of tangency.
- Prove that two chords equally distant from the center of a circle are congruent.
- Prove that if one side of a triangle inscribed in a circle is a diameter, then the triangle is a right triangle.
- Prove that if a radius of a circle is perpendicular to a chord of a circle, then the radius bisects the chord.

G.3.I Explain and perform constructions related to the circle.

Students perform constructions using straightedge and compass, paper folding, and dynamic geometry software. What is important is that students understand the mathematics and are able to justify each step in a construction.

Example:

- In each case, explain why the constructions work:
  a. Construct the center of a circle from two chords.
  b. Construct a circumscribed circle for a triangle.
  c. Inscribe a circle in a triangle.

Performance Expectation *Students are expected to:*		***Explanatory Comments and Examples***
G.3.J	Describe prisms, pyramids, parallelepipeds, tetrahedra, and regular polyhedra in terms of their faces, edges, vertices, and properties.	Examples: • Given the number of faces of a regular polyhedron, derive a formula for the number of vertices. • Describe symmetries of three-dimensional polyhedra and their two-dimensional faces. • Describe the lateral faces that are required for a pyramid to be a right pyramid with a regular base. Describe the lateral faces required for an oblique pyramid that has a regular base.
G.3.K	Analyze cross-sections of cubes, prisms, pyramids, and spheres and identify the resulting shapes.	Examples: • Start with a regular tetrahedron with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the tetrahedron is a square. Find the area of the square. (Requires no pencil or paper.) • Start with a cube with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the cube is a regular hexagon. Find the area of the hexagon. • Start with a cube with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the cube is a rectangle that is not a face and contains four of the vertices. Find the area of the rectangle. • Which has the larger area, the above rectangle or the above hexagon?

Geometry

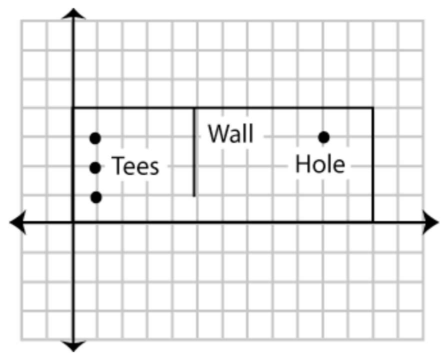
G.4. Core Content: *Geometry in the coordinate plane* *(Geometry/Measurement, Algebra)*

Students make connections between geometry and algebra by studying geometric properties and attributes that can be represented on the coordinate plane. They use the coordinate plane to represent situations that are both purely mathematical and that arise in applied contexts. In this way, they use the power of algebra to solve problems about shapes and space.

Performance Expectation *Students are expected to:*	**Explanatory Comments and Examples**
G.4.A Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line.	Examples: • Write an equation for the perpendicular bisector of a given line segment. • Determine the equation of a line through the points (5, 3) and (5, -2). • Prove that the slopes of perpendicular lines are negative inverses of each other.
G.4.B Determine the coordinates of a point that is described geometrically.	Examples: • Determine the coordinates for the midpoint of a given line segment. • Given the coordinates of three vertices of a parallelogram, determine all possible coordinates for the fourth vertex. • Given the coordinates for the vertices of a triangle, find the coordinates for the center of the circumscribed circle and the length of its radius.
G.4.C Verify and apply properties of triangles and quadrilaterals in the coordinate plane.	Examples: • Given four points in a coordinate plane that are the vertices of a quadrilateral, determine whether the quadrilateral is a rhombus, a square, a rectangle, a parallelogram, or none of these. Name all the classifications that apply. • Given a parallelogram on a coordinate plane, verify that the diagonals bisect each other. • Given the line with *y*-intercept 4 and *x*-intercept 3, find the area of a square that has one corner on the origin and the opposite corner on the line described.

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples

- Below is a diagram of a miniature golf hole as drawn on a coordinate grid. The dimensions of the golf hole are 4 feet by 12 feet. Players must start their ball from one of the three tee positions, located at (1, 1), (1, 2), or (1, 3). The hole is located at (10, 3). A wall separates the tees from the hole. At which tee should the ball be placed to create the shortest "hole-in-one" path? Sketch the intended path of the ball, find the distance the ball will travel, and describe your reasoning. (Assume the diameters of the golf ball and the hole are 0 for the mathematical model.)



G.4.D Determine the equation of a circle that is described geometrically in the coordinate plane and, given equations for a circle and a line, determine the coordinates of their intersection(s).

Examples:

- Write an equation for a circle with a radius of 2 units and center at (1, 3).
- Given the circle $x^2 + y^2 = 4$ and the line $y = x$, find the points of intersection.
- Write an equation for a circle given a line segment as a diameter.
- Write an equation for a circle determined by a given center and tangent line.

Geometry

G.5. Core Content: *Geometric transformations* *(Geometry/Measurement)*

Students continue their study of geometric transformations, focusing on the effect of such transformations and the composition of transformations on the attributes of geometric figures. They study techniques for establishing congruence and similarity by means of transformations.

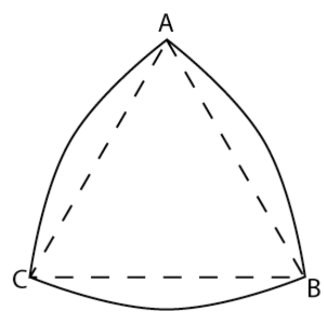
Performance Expectation *Students are expected to:*		***Explanatory Comments and Examples***
G.5.A	Sketch results of transformations and compositions of transformations for a given two-dimensional figure on the coordinate plane, and describe the rule(s) for performing translations or for performing reflections about the coordinate axes or the line $y = x$.	Transformations include translations, rotations, reflections, and dilations. Example: • Line *m* is described by the equation $y = 2x + 3$. Graph line *m* and reflect line *m* across the line $y = x$. Determine the equation of the image of the reflection. Describe the relationship between the line and its image.
G.5.B	Determine and apply properties of transformations.	Students make and test conjectures about compositions of transformations and inverses of transformations, the commutativity and associativity of transformations, and the congruence and similarity of two-dimensional figures under various transformations. Examples: • Identify transformations (alone or in composition) that preserve congruence. • Determine whether the composition of two reflections of a line is commutative. • Determine whether the composition of two rotations about the same point of rotation is commutative. • Find a rotation that is equivalent to the composition of two reflections over intersecting lines. • Find the inverse of a given transformation.
G.5.C	Given two congruent or similar figures in a coordinate plane, describe a composition of translations, reflections, rotations, and dilations that superimposes one figure on the other.	Examples: • Find a sequence of transformations that superimposes the segment with endpoints (0, 0) and (2, 1) on the segment with endpoints (4, 2) and (3, 0). • Find a sequence of transformations that superimposes the triangle with vertices (0, 0), (1, 1), and (2, 0) on the triangle with vertices (0, 1), (2, -1), and (0, -3).

Performance Expectation *Students are expected to:*		*Explanatory Comments and Examples*
G.5.D	Describe the symmetries of two-dimensional figures and describe transformations, including reflections across a line and rotations about a point.	Although the expectation only addresses two-dimensional figures, classroom activities can easily extend to three-dimensional figures. Students can also describe the symmetries, reflections across a plane, and rotations about a line for three-dimensional figures.

Geometry

G.6. Additional Key Content *(Measurement)*

Students extend and formalize their work with geometric formulas for perimeter, area, surface area, and volume of two- and three-dimensional figures, focusing on mathematical derivations of these formulas and their applications in complex problems. They use properties of geometry and measurement to solve problems in purely mathematical as well as applied contexts. Students understand the role of units in measurement and apply what they know to solve problems involving derived measures like speed or density. They understand that all measurement is approximate and specify precision in measurement problems.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
G.6.A Derive and apply formulas for arc length and area of a sector of a circle.	Example: • Find the area and perimeter of the Reauleaux triangle below. The Reuleaux triangle is constructed with three arcs. The center of each arc is located at the vertex of an equilateral triangle. Each arc extends between the two opposite vertices of the equilateral triangle. The figure below is a Reuleaux triangle that circumscribes equilateral triangle ABC. ΔABC has side length of 5 inches. $\overset{\frown}{AB}$ has center C, $\overset{\frown}{BC}$ has center A, and $\overset{\frown}{CA}$ has center B, and all three arcs have the same radius equal to the length of the sides of the triangle. 
G.6.B Analyze distance and angle measures on a sphere and apply these measurements to the geometry of the earth.	Examples: • Use a piece of string to measure the distance between two points on a ball or globe; verify that the string lies on an arc of a great circle. • On a globe, show with examples why airlines use polar routes instead of flying due east from Seattle to Paris. • Show that the sum of the angles of a triangle on a sphere is greater than 180 degrees.

Performance Expectation *Students are expected to:*		*Explanatory Comments and Examples*
G.6.C	Apply formulas for surface area and volume of three-dimensional figures to solve problems.	Problems include those that are purely mathematical as well as those that arise in applied contexts. Three-dimensional figures include right and oblique prisms, pyramids, cylinders, cones, spheres, and composite three-dimensional figures. Example: • As Pam scooped ice cream into a cone, she began to formulate a geometry problem in her mind. If the ice cream was perfectly spherical with diameter 2.25" and sat on a geometric cone that also had diameter 2.25" and was 4.5" tall, would the cone hold all the ice cream as it melted (without her eating any of it)? She figured the melted ice cream would have the same volume as the unmelted ice cream. Find the solution to Pam's problem and justify your reasoning. • A rectangle is 5 inches by 10 inches. Find the volume of a cylinder that is generated by rotating the rectangle about the 10-inch side.
G.6.D	Predict and verify the effect that changing one, two, or three linear dimensions has on perimeter, area, volume, or surface area of two- and three-dimensional figures.	The emphasis in high school should be on verifying the relationships between length, area, and volume and on making predictions using algebraic methods. Example: • What happens to the volume of a rectangular prism if four parallel edges are doubled in length? • The ratio of a pair of corresponding sides in two similar triangles is 5:3. The area of the smaller triangle is 108 in^2. What is the area of the larger triangle?
G.6.E	Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.	Example: • The U.S. Census Bureau reported a national population of 299,894,924 on its Population Clock in mid-October of 2006. One can say that the U.S. population is 3 hundred million (3×10^8) and be precise to one digit. Although the population had surpassed 3 hundred million by the end of that month, explain why 3×10^8 remained precise to one digit.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
G.6.F Solve problems involving measurement conversions within and between systems, including those involving derived units, and analyze solutions in terms of reasonableness of solutions and appropriate units.	This performance expectation is intended to build on students' knowledge of proportional relationships. Students should understand the relationship between scale factors and their inverses as they relate to choices about when to multiply and when to divide in converting measurements. Derived units include those that measure speed, density, flow rates, population density, etc. Example: • A digital camera takes pictures that are 3.2 megabytes in size. If the pictures are stored on a 1-gigabyte card, how many pictures can be taken before the card is full?

Geometry

***G.7. Core Processes:** Reasoning, problem solving, and communication*

Students formalize the development of reasoning in Geometry as they become more sophisticated in their ability to reason inductively and begin to use deductive reasoning in formal proofs. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students use a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. They use correct mathematical language, terms, symbols, and conventions as they address problems in Geometry and provide descriptions and justifications of solution processes. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways, and more and more occupations and fields of study rely on mathematics.

Performance Expectation

Students are expected to:

G.7.A Analyze a problem situation and represent it mathematically.

G.7.B Select and apply strategies to solve problems.

G.7.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.

G.7.D Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.

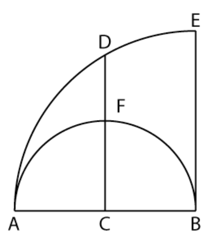
G.7.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.

G.7.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.

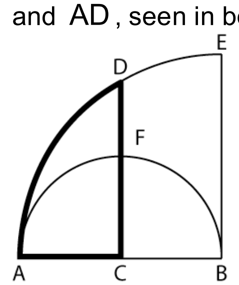
Explanatory Comments and Examples

Examples:

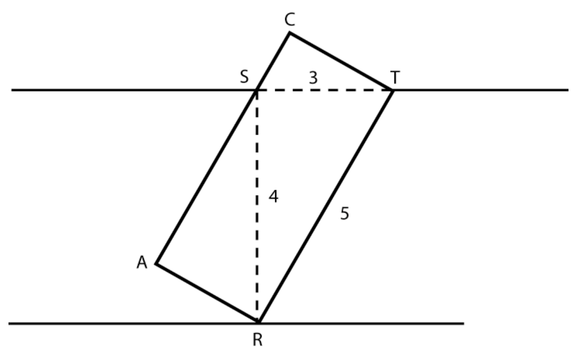
- $\overline{AB}$ is the diameter of the semicircle and the radius of the quarter circle shown in the figure below. $\overline{DC}$ is the perpendicular bisector of $\overline{AB}$.



Imagine all of the triangles formed by $\overline{AB}$ and any arbitrary point lying in the region bounded by $\overline{AC}$, $\overline{CD}$, and $\overset{\frown}{AD}$, seen in bold below.



Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
G.7.G Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.	Use inductive reasoning to make conjectures about what types of triangles are formed based upon the region where the third vertex is located. Use deductive reasoning to verify your conjectures.
G.7.H Use inductive reasoning to make conjectures, and use deductive reasoning to prove or disprove conjectures.	• Rectangular cartons that are 5 feet long need to be placed in a storeroom that is located at the end of a hallway. The walls of the hallway are parallel. The door into the hallway is 3 feet wide and the width of the hallway is 4 feet. The cartons must be carried face up. They may not be tilted. Investigate the width and carton top area that will fit through the doorway.

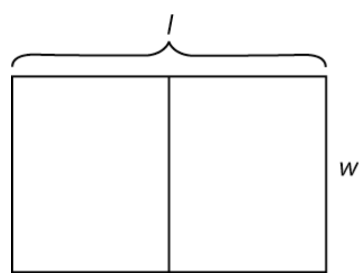


Generalize your results for a hallway opening of *x* feet and a hallway width of *y* feet if the maximum carton dimensions are *c* feet long and $x^2 + y^2 = c^2$.

Algebra 2

A2.1. Core Content: *Solving Problems*

The first core content area highlights the type of problems students will be able to solve by the end of Algebra 1I, as they extend their ability to solve problems with additional functions and equations. When presented with a word problem, students are able to determine which function or equation models the problem and use that information to solve the problem. They build on what they learned in Algebra I about linear and quadratic functions and are able to solve more complex problems. Additionally, students learn to solve problems modeled by exponential and logarithmic functions, systems of equations and inequalities, inverse variations, and combinations and permutations. Turning word problems into equations that can be solved is a skill students hone throughout Algebra II and subsequent mathematics courses.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.1.A Select and justify functions and equations to model and solve problems.	Examples: • A manufacturer wants to design a cylindrical soda can that will hold 500 milliliters (mL) of soda. The manufacturer's research has determined that an optimal can height is between 10 and 15 centimeters. Find a function for the radius in terms of the height, and use it to find the possible range of radius measurements in centimeters. Explain your reasoning. • Dawson wants to make a horse corral by creating a rectangle that is divided into 2 parts, similar to the following diagram. He has a 1200-foot roll of fencing to do the job. — What are the dimensions of the enclosure with the largest total area? — What function or equation best models this situation? 

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.1.B Solve problems that can be represented by systems of equations and inequalities.	Examples: • Mr. Smith uses the following formula to calculate students' final grades in his Algebra 1I class: $0.4E + 0.6T = C$, where E represents the score on the final exam, and T represents the average score of all tests given during the grading period. All tests and the final exam are worth a maximum of 100 points. The minimum passing score on tests, the final exam, and the course is 60. Determine the inequalities that describe the following situation and sketch a system of graphs to illustrate it. When necessary, round scores to the nearest tenth. — Is it possible for a student to have a failing test score average (i.e., $T < 60$ points) and still pass the course? — If you answered "yes," what is the minimum test score average a student can have and still pass the course? What final exam score is needed to pass the course with a minimum test score average? — A student has a particular test score average. How can (s)he figure out the minimum final exam score needed to pass the course? • Data derived from an experiment seems to be parabolic when plotted on a coordinate grid. Three observed data points are (2, 10), (3, 8), and (4, 4). Write a quadratic equation that passes through the points.
A2.1.C Solve problems that can be represented by quadratic functions, equations, and inequalities.	In addition to solving area and velocity problems by factoring and applying the quadratic formula to the quadratic equation, students use the vertex form of the equation to solve problems about maximums, minimums, and symmetry. Examples: • The Gateway Arch in St. Louis has a special shape called a catenary, which looks a lot like a parabola. It has a base width of 600 feet and is 630 feet high. Which is taller, this catenary arch or a parabolic arch that has the same base width but has a height of 450 feet at a point 150 feet from one of the pillars? What is the height of the parabolic arch?

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
	• Fireworks are launched upward from the ground with an initial velocity of 160 feet per second. The formula for vertical motion is $h(t) = 0.5at^2 + vt + s$, where the gravitational constant, a, is -32 feet per square second, v represents the initial velocity, and s represents the initial height. Time t is measured in seconds, and height h is measured in feet. For the ultimate effect, the fireworks must explode after they reach the maximum height. For the safety of the crowd, they must explode at least 256 ft. above the ground. The fuses must be set for the appropriate time interval that allows the fireworks to reach this height. What range of times, starting from initial launch and ending with fireworks explosion, meets these conditions?
A2.1.D Solve problems that can be represented by exponential and logarithmic functions and equations.	Examples: • If you need \$15,000 in 4 years to start college, how much money would you need to invest now? Assume an annual interest rate of 4% compounded monthly for 48 months. • The half-life of a certain radioactive substance is 65 days. If there are 4.7 grams initially present, how long will it take for there to be less than 1 gram of the substance remaining?
A2.1.E Solve problems that can be represented by inverse variations of the forms $f(x) = \frac{a}{x} + b$, $f(x) = \frac{a}{x^2} + b$, and $f(x) = \frac{a}{(bx+c)}$.	Examples: • At the You're Toast, Dude! toaster company, the weekly cost to run the factory is \$1400, and the cost of producing each toaster is an additional \$4 per toaster. — Find a function to represent the weekly cost in dollars, $C(x)$, of producing x toasters. Assume either unlimited production is possible or set a maximum per week. — Find a function to represent the total production cost per toaster for a week. — How many toasters must be produced within a week to have a total production cost per toaster of \$8? • A person's weight varies inversely as the square of his distance from the center of the earth. Assume the radius of the earth is 4000 miles. How much would a 200-pound man weigh — 1000 miles above the surface of the earth? — 2000 miles above the surface of the earth?

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.1.F Solve problems involving combinations and permutations.	Example: • The company Ali works for allows her to invest in her choice of 10 different mutual funds, 6 of which grew by at least 5% over the last year. Ali randomly selected 4 of the 10 funds in which to invest. What is the probability that 3 of Ali's funds grew by 5%? • Four points (*A*, *B*, *C*, and *D*) lie on one straight line, *n*, and five points (*E*, *F*, *G*, *H*, and *J)* lie on another straight line, *m*, that is parallel to *n*. What is the probability that three points, selected at random, will form a triangle?

Algebra 2

A2.2. Core Content: *Numbers, expressions, and operations* *(Numbers, Operations, Algebra)*

Students extend their understanding of number systems to include complex numbers, which they will see as solutions for quadratic equations. They grow more proficient in their use of algebraic techniques as they continue to use variables and expressions to solve problems. As problems become more sophisticated and the level of mathematics increases, so does the complexity of the symbolic manipulations and computations necessary to solve the problems. Students refine the foundational algebraic skills they need to be successful in subsequent mathematics courses.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.2.A Explain how whole, integer, rational, real, and complex numbers are related, and identify the number system(s) within which a given algebraic equation can be solved.	Example: • Within which number system(s) can each of the following be solved? Explain how you know. — $3x + 2 = 5$ — $x^2 = 1$ — $x^2 = \frac{1}{4}$ — $x^2 = 2$ — $x^2 = -2$ — $\frac{x}{7} = \pi$
A2.2.B Use the laws of exponents to simplify and evaluate numeric and algebraic expressions that contain rational exponents.	Examples: • Convert the following from a radical to exponential form or visa versa. — $24^{\frac{1}{3}}$ — $\sqrt[5]{16}$ — $\sqrt{x^2+1}$ — $\frac{x^2}{\sqrt{x}}$ • Evaluate $x^{-3/2}$ for $x = 27$

Performance Expectation	*Explanatory Comments and Examples*
Students are expected to:	
A2.2.C Add, subtract, multiply, divide, and simplify rational and more general algebraic expressions.	In the same way that integers were extended to fractions, polynomials are extended to rational expressions. Students must be able to perform the four basic arithmetic operations on more general expressions that involve exponentials. The binomial theorem is useful when raising expressions to powers, such as $(x + 3)^5$. Examples: • $\frac{x+1}{(x+1)^2}-\frac{3x-3}{x^2-1}$ • Divide $\frac{(x+2)^{3/2}}{x+1}$ by $\frac{x+2}{x^2-1}$

Algebra 2

A2.3. Core Content: *Quadratic functions and equations* *(Algebra)*

As students continue to solve quadratic equations and inequalities in Algebra 2, they encounter complex roots for the first time. They learn to translate between forms of quadratic equations, applying the vertex form to evaluate maximum and minimum values and find symmetry of the graph, and they learn to identify which form should be used in a particular situation. This opens up a whole range of new problems students can solve using quadratics. These algebraic skills are applied in subsequent high school mathematics and statistics courses.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.3.A Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.	Students translate among forms of a quadratic function to convert to one that is appropriate—e.g., vertex form—to solve specific problems. Students learn about the advantages of the standard form ($f(x) = ax^2 + bx + c$), the vertex form ($f(x) = a(x – h)^2 + d$), and the factored form ($f(x) = a(x – r)(x – s)$). They produce the vertex form by completing the square on the function in standard form, which allows them to see the symmetry of the graph of a quadratic function as well as the maximum or minimum. This opens up a whole range of new problems students can solve using quadratics. Students continue to find the solutions of the equation, which in Algebra 2 can be either real or complex. Example: • Find the minimum, the line of symmetry, and the roots for the graphs of each of the following functions: $f(x) = x^2 – 4x + 3$ $f(x) = x^2 – 4x + 4$ $f(x) = x^2 – 4x + 5$
A2.3.B Determine the number and nature of the roots of a quadratic function.	Students should be able to recognize and interpret the discriminant. Students should also be familiar with the Fundamental Theorem of Algebra, i.e., that all polynomials, not just quadratics, have roots over the complex numbers. This concept becomes increasingly important as students progress through mathematics. Example: • For what values of a does $x^2 – 6x + a$ have 2 real roots, 1 real root, and no real roots?

Performance Expectation Students are expected to:	*Explanatory Comments and Examples*
A2.3.C Solve quadratic equations and inequalities, including equations with complex roots.	Students solve equations that are not easily factored by completing the square and by using the quadratic formula.

Examples:

- $x^2 - 10x + 34 = 0$
- $3x^2 + 10 = 4x$
- Wile E. Coyote launches an anvil from 180 feet above the ground at time $t = 0$. The equation that models this situation is given by $h = -16t^2 + 96t + 180$, where t is time measured in seconds and h is height above the ground measured in feet.
  a. What is a reasonable domain restriction for t in this context?
  b. Determine the height of the anvil two seconds after it was launched.
  c. Determine the maximum height obtained by the anvil.
  d. Determine the time when the anvil is more than 100 feet above ground.
- Farmer Helen wants to build a pigpen. With 100 feet of fence, she wants a rectangular pen with one side being a side of her existing barn. What dimensions should she use for her pigpen in order to have the maximum number of square feet?

Algebra 2

A2.4. Core Content: *Exponential and logarithmic functions and equations* *(Algebra)*

Students extend their understanding of exponential functions from Algebra 1 with an emphasis on inverse functions. This leads to a natural introduction of logarithms and logarithmic functions. They learn to use the basic properties of exponential and logarithmic functions, graphing both types of function to analyze relationships, represent and model problems, and answer questions. Students employ these functions in many practical situations, such as applying exponential functions to determine compound interest and applying logarithmic functions to determine the pH of a liquid.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.4.A Know and use basic properties of exponential and logarithmic functions and the inverse relationship between them.	Examples: • Given $f(x) = 4^x$, write an equation for the inverse of this function. Graph the functions on the same coordinate grid. — Find $f(-3)$. — Evaluate the inverse function at 7. • Derive the formulas: — $\log_b a \cdot \log_a b = 1$ — $\log_a N = \log_b N \cdot \log_a b$ • Find the exact value of x in: — $\log_x 16 = \frac{4}{3}$ — $\log_3 81 = x$ • Solve for y in terms of x: — $\log_a \frac{y}{x} = x$ — $100 = x \cdot 10^y$
A2.4.B Graph an exponential function of the form $f(x) = ab^x$ and its inverse logarithmic function.	Students expand on the work they did in Algebra 1 to functions of the form $y = ab^x$. Although the concept of inverses is not fully developed until Precalculus, there is an emphasis in Algebra 2 on students recognizing the inverse relationship between exponential and logarithmic functions and how this is reflected in the shapes of the graphs. Example: • Find the equation for the inverse function of $y = 3^x$. Graph both functions. What characteristics of each of the graphs indicate they are inverse functions?

Performance Expectation *Students are expected to:*	*Explanatory Comments and Examples*
A2.4.C Solve exponential and logarithmic equations.	Example: • A recommended adult dosage of the cold medication NoMoreFlu is 16 mL. NoMoreFlu causes drowsiness when there are more than 4 mL in one's system, making it unsafe to drive, operate machinery, etc. The manufacturer wants to print a warning label telling people how long they should wait after taking NoMoreFlu for the drowsiness to pass. If the typical metabolic rate is such that one quarter of the NoMoreFlu is lost every four hours, and a person takes the full dosage, how long should adults wait after taking NoMoreFlu to ensure that there will be — Less than 4 mL of NoMoreFlu in their system? — Less than 1 mL in their system? — Less than 0.1 mL in their system? • Solve for x in $256 = 2^{x^2-1}$. • Solve for x in $\log_5(x - 4) = 3$

Algebra 2

***A2.5. Core Content:** Additional functions and equations* *(Algebra)*

Students learn about additional classes of functions including square root, cubic, logarithmic, and those involving inverse variation. Students plot points and sketch graphs to represent these functions and use algebraic techniques to solve related equations. In addition to studying the defining characteristics of each of these classes of functions, students gain the ability to construct new functions algebraically and using transformations. These extended skills and techniques serve as the foundation for further study and analysis of functions in subsequent mathematics courses.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.5.A Construct new functions using the transformations $f(x - h)$, $f(x) + k$, $cf(x)$, and by adding and subtracting functions, and describe the effect on the original graph(s).	Students perform simple transformations on functions, including those that contain the absolute value of expressions, quadratic expressions, square root expressions, and exponential expressions, to make new functions. Examples: • What sequence of transformations changes $f(x) = x^2$ to $g(x) = -5(x - 3)^2 + 2$? • Carly decides to earn extra money by making glass bead bracelets. She purchases tools for $40. Elastic bead cord for each bracelet costs $0.10. Glass beads come in packs of 10 beads, and one pack has enough beads to make one bracelet. Base price for the beads is $2.00 per pack. For each of the first 100 packs she buys, she gets $0.01 off each of the packs. (For example, if she purchases three packs, each pack costs $1.97 instead of $2.00.) Carly plans to sell each bracelet for $4.00. Assume Carly will make a maximum of 100 bracelets. — Find a function $C(b)$ that describes Carly's costs. — Find a function $R(b)$ that describes Carly's revenue. Carly's profit is described by $P(b) = R(b) - C(b)$. — Find $P(b)$. — What is the minimum number of bracelets that Carly must sell in order to make a profit? — To make a profit of $100?

Performance Expectation *Students are expected to:*	*Explanatory Comments and Examples*
A2.5.B Plot points, sketch, and describe the graphs of functions of the form $f(x) = a\sqrt{x-c} + d$, and solve related equations.	Students solve algebraic equations that involve the square root of a linear expression over the real numbers. Students should be able to identify extraneous solutions and explain how they arose. Students should view the function $g(x) = \sqrt{x}$ as the inverse function of $f(x) = x^2$, recognizing that the functions have different domains for x greater than or equal to 0. Example: • Analyze the following equations and tell what you know about the solutions. Then solve the equations. __ $2\sqrt{x+5} = 7$ __ $\sqrt{5x-6} = -2$ __ $\sqrt{2x+15} = x$ __ $\sqrt{2x-5} = x+7$
A2.5.C Plot points, sketch, and describe the graphs of functions of the form $f(x) = \frac{a}{x} + b$, $f(x) = \frac{a}{x^2} + b$, and $f(x) = \frac{a}{(bx+c)}$, and solve related equations.	Examples: • Sketch the graphs of the four functions $f(x) = \frac{a}{x^2} + b$ when a = 4 and 8 and b = 0 and 1. • Sketch the graphs of the four functions $f(x) = \frac{4}{(bx+c)}$ when b = 1 and 4 and c = 2 and 3.
A2.5.D Plot points, sketch, and describe the graphs of cubic polynomial functions of the form $f(x) = ax^3 + d$ as an example of higher order polynomials and solve related equations.	Examples: • Solve for x in $60 = -2x^3 + 6$.

Algebra 2

A2.6. Core Content: *Probability, data, and distributions* *(Data/Statistics/Probability)*

Students formalize their study of probability, computing both combinations and permutations to calculate the likelihood of an outcome in uncertain circumstances and applying the binominal theorem to solve problems. They extend their use of statistics to graph bivariate data and analyze its shape to make predictions. They calculate and interpret measures of variability, confidence intervals, and margins of error for population proportions. Dual goals underlie the content in the section: students prepare for the further study of statistics and become thoughtful consumers of data.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.6.A Apply the fundamental counting principle and the ideas of order and replacement to calculate probabilities in situations arising from two-stage experiments (compound events).	
A2.6.B Given a finite sample space consisting of equally likely outcomes and containing events A and B, determine whether A and B are independent or dependent, and find the conditional probability of A given B.	Example: • What is the probability of drawing a heart from a standard deck of cards on a second draw, given that a heart was drawn on the first draw and not replaced?
A2.6.C Compute permutations and combinations, and use the results to calculate probabilities.	Example: • Two friends, Abby and Ben, are among five students being considered for three student council positions. If each of the five students has an equal likelihood of being selected, what is the probability that Abby and Ben will both be selected?
A2.6.D Apply the binomial theorem to solve problems involving probability.	The binominal theorem is also applied when computing with polynomials. Examples: • Use Pascal's triangle and the binomial theorem to find the number of ways six objects can be selected four at a time. • In a survey, 33% of adults reported that they preferred to get the news from newspapers rather than television. If you survey 5 people, what is the probability of getting exactly 2 people who say they prefer news from the newspaper? — Write an equation that can be used to solve the problem. — Create a histogram of the binomial distribution of the probablity of getting 0 through 5 responders saying they prefer the newspaper.

Performance Expectation *Students are expected to:*	*Explanatory Comments and Examples*
A2.6.E Determine if a bivariate data set can be better modeled with an exponential or a quadratic function and use the model to make predictions.	In high school, determining a formula for a curve of best fit requires a graphing calculator or similar technological tool.
A2.6.F Calculate and interpret measures of variability and standard deviation and use these measures and the characteristics of the normal distribution to describe and compare data sets.	Students should be able to identify unimodality, symmetry, standard deviation, spread, and the shape of a data curve to determine whether the curve could reasonably be approximated by a normal distribution. Given formulas, student should be able to calculate the standard deviation for a small data set, but calculators ought to be used if there are very many points in the data set. It is important that students be able to describe the characteristics of the normal distribution and identify common examples of data that are and are not reasonably modeled by it. Common examples of distributions that are approximately normal include physical performance measurements (e.g., weightlifting, timed runs), heights, and weights. Apply the Empirical Rule (68–95–99.7 Rule) to approximate the percentage of the population meeting certain criteria in a normal distribution. Example: • Which is more likely to be affected by an outlier in a set of data, the interquartile range or the standard deviation?
A2.6.G Calculate and interpret margin of error and confidence intervals for population proportions.	Students will use technology based on the complexity of the situation. Students use confidence intervals to critique various methods of statistical experimental design, data collection, and data presentation used to investigate important problems, including those reported in public studies. Example: • In 2007, 400 of the 500 10th graders in Local High School passed the WASL. In 2008, 375 of the 480 10th graders passed the test. The Local Gazette headline read "10th Grade WASL Scores Decline in 2008!" In response, the Superintendent of Local School District wrote a letter to the editor claiming that, in fact, WASL performance was not significantly lower in 2008 than it was in 2007. Who is correct, the Local Gazette or the Superintendent?

Performance Expectation	*Explanatory Comments and Examples*
Students are expected to:	
	Use mathematics to find the margin of error to justify your conclusion. (Formula for the margin of error (E): $E = z_c\sqrt{\frac{p(1-p)}{n}}$; $z_{95} = 1.96$, where n is the sample size, p is the proportion of the sample with the trait of interest, c is the confidence level, and z_c is the multiplier for the specified confidence interval.)

Algebra 2

A2.7. Additional Key Content *(Algebra)*

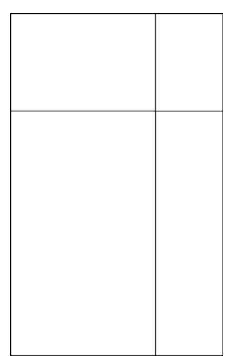
Students study two important topics here. First, they extend their ability to solve systems of two equations in two variables to solving systems of three equations in three variables, which leads to the full development of matrices in Precalculus. Second, they formalize their work with series as they learn to find the terms and partial sums of arithmetic series and the terms and partial and infinite sums of geometric series. This conceptual understanding of series lays an important foundation for understanding calculus.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.7.A Solve systems of three equations with three variables.	Students solve systems of equations using algebraic and numeric methods. Examples: • Jill, Ann, and Stan are to inherit $20,000. Stan is to get twice as much as Jill, and Ann is to get twice as much as Stan. How much does each get? • Solve the following system of equations. $2x - y - z = 7$ $3x + 5y + z = -10$ $4x - 3y + 2z = 4$
A2.7.B Find the terms and partial sums of arithmetic and geometric series and the infinite sum for geometric series.	Students build on the knowledge gained in Algebra 1 to find specific terms in a sequence and to express arithmetic and geometric sequences in both explicit and recursive forms. Examples: • A ball is dropped from a height of 10 meters. Each time it hits the ground, it rebounds $\frac{3}{4}$ of the distance it has fallen. What is the total sum of the distances it falls and rebounds before coming to rest? • Show that the sum of the first 10 terms of the geometric series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$ is twice the sum of the first 10 terms of the geometric series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots$

Algebra 2

A2.8. Core Processes: *Reasoning, problem solving, and communication*

Students formalize the development of reasoning at high school as they use algebra and the properties of number systems to develop valid mathematical arguments, make and prove conjectures, and find counterexamples to refute false statements using correct mathematical language, terms, and symbols in all situations. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students formalize a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.8.A Analyze a problem situation and represent it mathematically.	Examples:
A2.8.B Select and apply strategies to solve problems.	• Show that $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$, for all positive real values of a and b.
A2.8.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.	• Show that the product of two odd numbers is always odd.
A2.8.D Generalize a solution strategy for a single problem to a class of related problems and apply a strategy for a class of related problems to solve specific problems.	• Leo is painting a picture on a canvas that measures 32 inches by 20 inches. He has divided the canvas into four different rectangles, as shown in the diagram. 
A2.8.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.	He would like the upper right corner to be a rectangle that has a length 1.6 times its width. Leo wants the area of the larger rectangle in the lower left to be at least half the total area of the canvas.
A2.8.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.	

Performance Expectation *Students are expected to:*	*Explanatory Comments and Examples*

A2.8.G Use inductive reasoning and the properties of numbers to make conjectures, and use deductive reasoning to prove or disprove conjectures.

A2.8.H Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.

Describe all the possibilities for the dimensions of the upper right rectangle to the nearest hundredth, and explain why the possibilities are valid.

If Leo uses the largest possible dimensions for the smaller rectangle:

— What will the dimensions of the larger rectangle be?

— Will the larger rectangle be similar to the rectangle in the upper right corner? Why or why not?

— Is the original canvas similar to the rectangle in the upper right corner?

(A rectangle whose length and width are in the ratio $\frac{1+\sqrt{5}}{2}$ (approximately equal to 1.6) is called a "golden rectangle" and is often used in art and architecture.)

- A relationship between variables can be represented with a table, a graph, an equation, or a description in words.

  — How can you decide from a table whether a relationship is linear, quadratic, or exponential?

  — How can you decide from a graph whether a relationship is linear, quadratic, or exponential?

  — How can you decide from an equation whether a relationship is linear, quadratic, or exponential?