A Report to the Washington State Board of Education:

Follow-Up to *Review and Recommendations*

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Last summer, Strategic Teaching completed an independent review of Washington's K–12 mathematics standards. We presented our findings to the Washington State Board of Education (SBE) in late August along with seven recommendations for strengthening the state's mathematics standards.

While we noted a number of good qualities in the standards, we were quite blunt in our assessment of where Washington's standards did not measure up to best practice in this country or internationally. We said:

> *. . . compared to the standards of key states and high achieving countries, Washington is not expecting enough of its students. There is insufficient emphasis on core mathematical content. Some math should be taught earlier in a student's schooling, and some crucial math is missing completely. Simply put, Washington is not focused enough on the important fundamental content topics in mathematics.*

Washington took the findings and recommendations to heart. In the past several months, the Office of Superintendent of Public Instruction (OSPI), working in collaboration with the Charles A. Dana Center at the University of Texas at Austin, has crafted the new *Washington K–12 Mathematics Standards*. These were submitted to the Washington legislature on January 31, 2008.

At SBE's request, we examined the degree to which the new standards follow our seven recommendations. The purpose of this report is not to evaluate the standards, but to determine if they have met the recommendations of the first report.

The answer is a conditional yes. In grades K–8, the new mathematics standards are vastly superior in every way to the Grade Level Expectations that we initially examined. In

general, the recommendations were followed. In places where they were not, most issues are relatively easy to fix.

The new standards at the high school level are more problematic.

Each of the seven recommendations will be discussed in turn, and the discussion will include suggestions that would bring the new standards into alignment with the recommendations from our original report.

We also discuss below significant issues of the high school level standards that cut across a number of the recommendations. We have put these high school issues after the discussion of Recommendation 7.

Recommendation 1: Set higher expectations for Washington's students by fortifying content and increasing rigor.

At grades K–8, the *Washington K–12 Mathematics Standards* document includes rigorous grade-level mathematics content similar to the content required in other high-achieving states and countries. Items such as fractions, place value and word problems that were identified in *Review and Recommendations* as areas of concern have been addressed.

Washington has replaced the verb "understand" with demanding verbs that require students to understand *and* to do something with that understanding. Washington now asks its students to differentiate, represent, evaluate and much more.

Recommendation 2: Make clear the importance of all aspects of mathematics: mathematics content, including the standard algorithms; the conceptual understanding of the content; and the application of mathematical processes within the content.[1]

The *Washington K–12 Mathematics Standards* is a better-balanced document than the one it replaces. Mathematical

[1] Washington State defines the following process strands: communicating, reasoning, problem solving and making connections with mathematical content.

content, conceptual understanding and the application of mathematical processes are all evident.

One strength of Washington's previous document was its development of the mathematical processes. The new standards both better develop the other aspects of mathematics and increase the sophistication of the process standards. Not only are these ideas more succinct, but they are better woven into the content standards.

Nonetheless, there continues to be some ambivalence around mathematics facts and between standard algorithms and conceptual understanding.

First, mathematics facts: What we believe Washington wants for its students is that they know addition, subtraction, multiplication and division facts. "Know" is as impossible to see as "understand," but it may best be described in this case as "instantly recall." The language in Washington's new standards comes so close in grades 1 and 2 with "quickly recall" that we wouldn't argue. The specificity of the language deteriorates to "demonstrate mastery" in:

4.1.A Demonstrate mastery of multiplication and related division facts through 10 x 10 and use them to solve problems.

Mastery is ambiguous, and this should be changed to either "instantly recall" or "quickly recall".

The new document is much more clear than its predecessor about the importance of standard algorithms. With very small changes, the new standards will meet the recommendations. The document uses the plural — *algorithms* — in a way that suggests there are multiple, standard, equally useful procedures for multiplication and division. There are not. The "s" needs to be removed in the following standards:

4.1.E Multiply up to three-digit by two-digit numbers using the standard algorithms.

5.1.C Divide three- or four-digit numbers by two digit numbers, with and without remainders, using the standard algorithms.

It may sound as though we are quibbling over an "s" but this is really about making sure students learn, practice, and have fluency with standard procedures.

Additionally, adding "s" to algorithm in these standards could be interpreted to mean that students must know *the* standard algorithm *and* other algorithms equally well.

We believe it is important to let students look for multiple ways to solve problems and to generalize those approaches. Doing so builds understanding and is good teaching. But, in the end, students need to know and practice to fluency the commonly used, standard algorithm and we state that explicitly in *Review and Recommendations.* In 3.4.D below, fluency has been replaced with efficient, which does not have the same meaning. In 4.1.E and 5.1.C above, even efficiency is missing. Fluency with the standard algorithms is essential and these three standards should incorporate it.

3.4.D Add and subtract whole numbers efficiently using the standard algorithms and solve addition and subtraction word problems

Recommendation 3: Identify those topics that should be taught for extended periods at each grade level and better show how topics develop over grade levels.

The *Washington K–12 Mathematics Standards* document embraces this recommendation. There may be those who would identify different key content or who argue about the amount or placement of some of the additional content. That is beyond the scope of this review. We applaud the limited number of Core Content topics at each grade level, the clearly defined Additional Key Content and the Core Processes.

In addition, OSPI created strand documents that show how Numbers; Operations; Geometry; Measurement; Algebra; Data Analysis, Statistics and Probability; and Reasoning, Problem Solving and Communication develop throughout the K–8 grades.

Recommendation 4: Increase the clarity, specificity and measurability of the Grade Level Expectations.

Washington's previous Grade Level Expectations were broadly written and depended on the ambiguous verb "understand" to describe what students were to do. We recommended that the parameters of the content be defined and that "understand" be replaced with observable, action verbs that clarified what students should be able to do with the content. Additionally, we suggested that sample problems be included as needed for clarity.

The new standards do this, as shown by the following examples, which are accompanied by sample problems in the standards document:

6.1.D Multiply and divide non-negative fractions using the standard algorithms.

8.2.C Represent and explain the effect of one or more translations, rotations, reflections, or dilations (centered at the origin) on a geometric figure on a coordinate plane.

However, there are a number of places in the new document where there is inappropriate ambiguity, where the writers did not impose limits. For example:

F.4.B Sketch the graph of a quadratic function, describe the relationship between the characteristics of the graph and its parameters, and translate among various algebraic forms.

This standard has a great beginning, but rather than "various algebraic forms," we suggest listing the only forms that exist — the standard form, vertex form and factored form — each of which is mathematically important.

Additionally, there are dozens of places in which the new document uses the conjunction "or" in a way that muddles the clarity, measurability and specificity of a standard. This is best illustrated by example:

4.2.C Convert between mixed numbers and improper fractions using words, numbers, pictures, **or** physical materials.

5.2.A Represent addition and subtraction of fractions and mixed numbers using words, numbers, pictures, **or** physical materials, and translate among representations.

In the case of 4.2.C, the language suggests that students be allowed to choose to learn to convert between mixed numbers and fractions using only some of the ways listed in the standard. In the case of 5.2.A, the language suggests that a student might choose to learn to add and subtract fractions only by using pictures and physical materials and not using numbers, but that the student would still need to be able to translate among all of the listed representations of addition and subtraction of fractions.

We suspect that the writers of the new document want students to be able to convert or represent in all of these ways, but they do not want to suggest that every method be used in every situation. Replacing "or" with "and" does not say the student must do it every time, just that he or she can use each method when needed.

Although that is one simple way to fix the problem, there are others. What is important is that these problems are addressed.

Recommendation 5: Write Essential Academic Learning Requirements that restructure the standards document to clarify grade-level content priorities and that reflect both the conceptual and procedural sides of mathematics.

This recommendation, which is about the topics that structure the document, is met. The *Washington K–12 Mathematics Standards* document bears little resemblance to its predecessor. The confusing, redundant levels of components and subcomponents are gone. At the K–8 grade-level priorities are clearly identified, and those priorities reflect the importance of both understanding and doing mathematics.

Recommendation 6: Create a standards document that is easily used by most people.

The *Washington K–12 Mathematics Standards* document meets this recommendation for the first nine years of school, but it falls far short at the high school level, giving little guidance to high school teachers, students or their parents.

We say more about high school below.

Recommendation 7: Create small, expert Standards Revision Teams for each grade band and systematically collect feedback on the revised standards.

Although OSPI chose to work with far larger groups than we recommended, they have done good work within a very short time frame.

There has been extensive outreach and multiple feedback opportunities. By all accounts, the feedback has been seriously considered.

Grades 9–12

Simply put, the high school document is not there yet.

The standards have been beefed up, the content has been clarified and clarifying examples have been added. The *Exhibitions of Learning* have been elevated to standards status and the multilayered format has been replaced. The content is mostly defined, and the verbs are actionable.

However, there are serious problems.

The high school document's organizing structure is not helpful to teachers, nor does it communicate well to parents and students. How would one begin to know what content would be included in Integrated Math I or Geometry?

The *Review and Recommendations* report listed specific content to be considered during the standards rewrite. This

was not intended to be an exhaustive list, but we hoped it would be taken seriously. That was not always the case.

The areas of concern we identified for all graduating students are generally well attended to in the *Washington K–12 Mathematics Standards* document.

Things get more complicated in grades 11 and 12, and some of the complexities are beyond OSPI's control.

Since we wrote *Review and Recommendations*, the State Board of Education has drafted a proposal that will require almost all students to have Algebra II or its equivalency in order to graduate[2]. It can be argued that, while much of the mathematics in Algebra II is appropriate for all students, there is a subset of that content that is hard to image being useful for students who are not headed toward a mathematics-intensive field. For example, conic sections have limited applicability. This makes writing high school standards more challenging.

Another factor with unknown implications is the Washington Assessment of Student Learning, or WASL. The WASL has been a high-stakes test pegged at grade 10 and containing mathematical content appropriate for all students. The standards from the new mathematics standards that the WASL will measure are currently undetermined.

That said, the current high school document is simply a catchall of the content a student should encounter in high school, with some, but not all, of the content that would prepare him or her for college included and identified.

Washington K-12 Mathematics Standards is missing much of the content identified in *Review and Recommendations* as

[2] The State Board of Education is proposing a draft rule which would require all students beginning with the class of 2013 to take a 3rd credit of math in high school either through a traditional math course or a Career and Technical Education course that is aligned with the math standards for Algebra II. However, a student who has completed two credits of math aligned to the high school mathematics and he or she has a defined career path, will have the opportunity to substitute another math course for Algebra II if they obtain parent/guardian and high school approval.

necessary for a student to be calculus ready in college. To remind the reader, this content was identified by reviewing the content contained in Washington College Readiness Standards, American Diploma Project Benchmarks, and the National Assessment for Education Progress and noting those topics that are present in at least two of the documents.

Since most students will now be taking some form of Algebra II, we would like to refine our recommendation from *Review and Recommendation.* In addition to all of the content necessary for graduation, *Washington's K-12 Mathematics Standards* include and identify all of the content necessary to enter college in a credit-bearing course and to enter college prepared to take Calculus.

We now see a document that needs to provide three kinds of information:

- Standards that everyone must master to graduate, pegged to Algebra II.
- College-readiness standards that prepare a student to enter college in a credit-bearing course.
- Calculus-ready standards that prepare a student to enter college and take Calculus as their first course.

The last two types of standards need to be included to communicate clearly so that teachers and students know what is necessary.

The current document provides one type of demarcation by italicizing the standards for students on the college-bound track. Another way would need to be found to identify Calculus-ready standards, unless this becomes obvious when the standards are organized by courses.

Summary

The K-8 standards come very close to meeting the recommendations and can do so with some relatively easy fixes. There is significantly more work to be done at the high school level.