HOMEWORK SOLUTIONS:

Section 4.9

13.)

\[ f(u) = \frac{u^4 + 3\sqrt{u}}{u^2} \]

Splitting the fraction,

\[ f(u) = \frac{u^4}{u^2} + \frac{3\sqrt{u}}{u^2} \]
\[ = u^2 + 3\frac{u^{\frac{1}{2}}}{u^2} \]
\[ = u^2 + 3u^{-\frac{1}{2}}. \]

So the antiderivative is

\[ F(u) = \frac{1}{3}u^3 + 3u^{-\frac{1}{2}} + C \]
\[ = \frac{1}{3}u^3 - 6u^{-\frac{1}{2}} + C \]

14.) By examining each part,

\[ F(u) = 3e^x + 7\tan x + C. \]

23.) Taking the anti-derivative of \( f'' \), we have

\[ f'(x) = 3x^2 + 4x^3 + C. \]

Taking the anti-derivative again, we have

\[ f(x) = x^3 + x^4 + Cx + D, \]

where \( C \) and \( D \) are constants.

30.) Here, we first take the antiderivative of \( f'(x) \):

\[ f(x) = 2x^4 + 6x^2 + 3x + C. \]

Now, we use the fact that \( f(1) = 6 \) to find \( C \):

\[ f(1) = 2(1) + 6(1) + 3(1) + C = 6 \]
\[ 11 + C = 6 \]
\[ C = -5. \]
So

\[ f(x) = 2x^4 + 6x^2 + 3x - 5. \]

31.) Here,

\[ f'(x) = x \frac{1}{2} (6 + 5x) = 6x \frac{1}{2} + 5x^\frac{3}{2}. \]

So

\[ f(x) = \frac{6}{2} x \frac{3}{2} + \frac{5}{2} x \frac{3}{2} + C = 4x \frac{3}{2} + 2x \frac{3}{2} + C. \]

Now, we use the fact that \( f(1) = 10 \) to find \( C \):

\[ f(1) = 4(1) + 2(1) + C = 10 \]

\[ C = 4. \]

So

\[ f(x) = 4x \frac{3}{2} + 2x \frac{3}{2} + 4. \]

44.) First, we take the anti-derivative of \( f''(t) \) to find \( f'(t) \):

\[ f'(t) = 2e^t - 3\cos t + C. \]

Next, we take the anti-derivative again to find \( f(t) \):

\[ f(t) = 2e^t - 3\sin t + Ct + D, \]

where \( C \) and \( D \) are both constants.

Now, to find \( C \) and \( D \), we use the initial conditions. First, let us use the condition \( f(0) = 0 \):

\[ f(0) = 2e^0 - 3\sin 0 + C(0) + D = 0 \]

\[ 2 + D = 0 \]

\[ D = -2. \]

So

\[ f(t) = 2e^t - 3\sin t + Ct - 2. \]

Now, we use the condition that \( f(\pi) = 0 \):

\[ f(\pi) = 2e^\pi - 3\sin \pi + C(\pi) - 2 = 0 \]

\[ 2e^\pi - 2 + C\pi = 0 \]

\[ C = \frac{2 - 2e^\pi}{\pi}. \]

Thus,

\[ f(t) = 2e^t - 3\sin t + \frac{2 - 2e^\pi}{\pi}t - 2. \]
Section 5.2

2.) We split the interval 0 to 3 into 6 intervals, each with size $\frac{1}{2}$. So

$$R_6 = \frac{1}{2} \left( f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2) + f\left(\frac{5}{2}\right) + f(3) \right)$$

$$= \frac{1}{2} \left( -\frac{3}{4} - 1 - \frac{3}{4} + 0 + \frac{5}{4} + 3 \right)$$

$$= \frac{1}{2} \left( \frac{7}{4} \right)$$

$$= \frac{7}{8}$$

5.) I think I meant to assign problem 4. Oops.

Well, whatever. For this one, the left endpoints give area 6 and the right ones give 4. See the diagram once I upload the diagram.

21.) Let’s split this into three intervals (the problem doesn’t specify, so I will). The total length of the region is 6, so each rectangle has length 2.

So

$$L_3 = 2(f(-1) + f(1) + f(3)) = 2(-2 + 4 + 10) = 24$$

and

$$R_3 = 2(f(1) + f(3) + f(5)) = 2(4 + 10 + 16) = 38.$$ 

24.) Let’s split this into 5 intervals of length 1 apiece:

$$L_5 = 1(f(0) + f(1) + f(2) + f(3) + f(4)) = 1(1 + 3 + 17 + 55 + 129) = 205$$

and

$$R_5 = 1(f(1) + f(2) + f(3) + f(4) + f(5)) = 1(3 + 17 + 55 + 129 + 251) = 455.$$
25.) Bringing the denominator up,
\[ \int_{1}^{2} \frac{3}{t^4} dt \]
\[ = \int_{1}^{2} 3t^{-4} dt \]
\[ = [-x^{-3}]_1^2 \]
\[ = 3 \left[ \frac{1}{2} \right]^2_1 \]
\[ = - \left[ \frac{1}{8} - 1 \right] \]
\[ = \frac{7}{8} \]

28.) Noting that \( x^{\sqrt{x}} = x^{\frac{x}{2}} \),
\[ \int_{0}^{1} (3 + x^{\frac{x}{2}}) dx \]
\[ = 3x + \left. x^{3} \right|_0^1 \]
\[ = 3 + \frac{2}{5} \]
\[ = \frac{17}{5} \]

32.) In this case,
\[ \int_{0}^{\frac{\pi}{4}} \sec \theta \tan \theta d\theta \]
\[ = \sec \theta \left|_0^{\frac{\pi}{4}} \right. \]
\[ = \sec \left( \frac{\pi}{4} \right) - \sec 0 \]
\[ = \frac{1}{\cos \left( \frac{\pi}{4} \right)} - \frac{1}{\cos 0} \]
\[ = \sqrt{2} - 1 \]
40.) Here, we split up the fraction:

\[
\int_{1}^{2} \left( \frac{4}{u^3} + \frac{u^2}{u^3} \right) du = \int_{1}^{2} \left( 4u^{-3} + \frac{1}{u} \right) du = -2u^{-2} + \ln |u| \bigg|_{1}^{2} = -\frac{2}{u^2} + \ln |u| \bigg|_{1}^{2} = -\frac{2}{4} + \ln |2| - [-2 + \ln 1] = \frac{3}{2} + \ln 2
\]

since \( \ln 1 = 0 \).
Section 5.4

14.) Splitting the integral up,

\[ \int (\csc^2 t - 2e^t)\,dt \]
\[ = \int \csc^2 t\,dt - \int 2e^t\,dt \]
\[ = -\cot x - 2e^t + C \]
13.) In the case of \[ \int \frac{dx}{5 - 3x}, \]
let us take \[ u = 5 - 3x. \]
So \[ du = -3dx \]
\[ dx = \frac{du}{-3} \]
Plugging back in:
\[
\int \frac{du}{-3(u)}
= -\frac{1}{3} \int \frac{du}{u}
= -\frac{1}{3} \ln|u| + C
= -\frac{1}{3} \ln|5 - 3x| + C.
\]

33.) Here, we note that \( \cot x \) appears to be the “inside” function. So we take \[ u = \cot x. \]
Then \[ du = -\csc^2 x \, dx \]
\[ dx = -\frac{du}{\csc^2 x} \]
Plugging back in:
\[
\int \sqrt{\cot x} \csc^2 x
= \int \sqrt{u} \csc^2 x \left( -\frac{du}{\csc^2 x} \right)
= -\int \sqrt{u} \, du
= -\frac{2}{3} u^{3/2} + C
= -\frac{2}{3} (\cot x)^{3/2} + C.
\]