HW 7 Solutions

1.) \( F(x) = 5 \left( \frac{2}{3} \right) x^{2} + \ln |x| + C \)
\[= \frac{10}{3} x^{2} + \ln |x| + C. \]

2.) \( G(x) = \frac{1}{2} e^{2x} + x+C \)

3.) \( H(t) = 4 \left( \frac{1}{4} \right) t^{-2} + 3(-1)t^{-1} + 2 \ln |t| + C \)
\[= -2t^{-2} - 3t^{-1} + 2 \ln |t| + C. \]

4.) First,
\[ f(x) = \frac{1}{3} x^{3} + \frac{3}{2} x^{2} + x + C. \]
Now, if we plug in 1 for \( x \), we get
\[ f(1) = \frac{1}{3} + \frac{3}{2} + 1 + C = 3. \]
Subtracting \( \frac{17}{6} \) from both sides, we get that \( C = \frac{1}{6}. \) So
\[ f(x) = \frac{1}{3} x^{3} + \frac{3}{2} x^{2} + x + \frac{1}{6}. \]

5.) In this case, we first take the antiderivative to find that
\[ f'(x) = x^{2} + 5x + C. \]
Plugging in 1 for \( x \), we have that
\[ f'(1) = 1 + 5 + C = 2. \]
Subtracting 6 from both sides, we get that \( C = -4. \) So
\[ f'(x) = x^{2} + 5x - 4. \]
Now, we take the antiderivative again, finding that
\[ f(x) = \frac{1}{3} x^{3} + \frac{5}{2} x^{2} - 4x + D. \]
Using the fact that \( f(1) = 3 \), we plug in to find that
\[ f(1) = \frac{1}{3} + \frac{5}{2} - 4 + D = 3. \]
Adding \( \frac{7}{6} \) to both sides gives us that \( D = \frac{25}{6}. \) Thus,
\[ f(x) = \frac{1}{3} x^{3} + \frac{5}{2} x^{2} - 4x + \frac{25}{6}. \]
6.) We know that acceleration is the second derivative of position. We’ll write position as \( p(t) \). Then
\[
p''(t) = -9.8.
\]
From this, we can first find velocity:
\[
p'(t) = -9.8t + C.
\]
Plugging in the fact that at one second, velocity is 156.8, we have that
\[
p'(1) = -9.8 + C = 156.8.
\]
Solving for \( C \), we find that \( C = 166.6 \). So
\[
p'(t) = -9.8t + 166.6.
\]
Now, we use this to find position. First,
\[
p(t) = -4.9t^2 + 166.6t + D.
\]
We must find \( D \). We know that at 2 seconds, the rocket has height 2000. So
\[
p(2) = -4.9(2)^2 + 166.6(2) + D = 2000,
\]
or
\[
314 + D = 2000,
\]
which means that \( D = 1686 \). So
\[
p(t) = -4.9t^2 + 166.6t + 1686.
\]

7.) Since we have a maximum number of fish, we use the equation
\[
P(t) = \frac{M}{1 + Be^{-Mkt}}.
\]
We know that \( M = 200 \) and \( k = .007 \). So
\[
P(t) = \frac{200}{1 + Be^{-200(.007)t}} = \frac{200}{1 + Be^{-1.4t}}.
\]
To find \( B \), we know that at time 0 (i.e. \( t = 0 \)), we have 150 fish. So
\[
P(0) = \frac{200}{1 + Be^0} = 150,
\]
or
\[
\frac{200}{1 + B} = 150.
\]
Multiplying both sides by $1 + B$ and dividing both sides by 150, we have
\[
\frac{200}{150} = 1 + B.
\]
Simplifying the left and subtracting 1 from both sides gives us that $B = \frac{1}{3}$. So
\[
P(t) = \frac{200}{1 + \left(\frac{1}{3}\right)e^{-1.4t}}.
\]
To find the number of fish after 2 or 50 hours, we simply plug in 2 or 50 for $t$:
\[
P(2) = \frac{200}{1 + \left(\frac{1}{3}\right)e^{-1.4(2)}} \approx 196,
\]
and
\[
P(50) = \frac{200}{1 + \left(\frac{1}{3}\right)e^{-1.4(50)}} \approx 200.
\]

8.) Here, we use the equation
\[
f(t) = \frac{P}{1 + Be^{-ct}}.
\]
Here, the population is 500,000. So
\[
f(t) = \frac{500000}{1 + Be^{-ct}}.
\]
To find our constants $B$ and $c$, we first use the information that at time 0, 100 people have seen the video:
\[
f(t) = \frac{500000}{1 + Be^{0}} = 100.
\]
As before, we multiply by $1 + B$ and divide by 100:
\[
\frac{500000}{100} = 1 + B.
\]
Solving for $B$ gives $B = 4999$. So
\[
f(t) = \frac{500000}{1 + 4999e^{-ct}}.
\]
Now, to find $c$, we use the fact that at time 3, 500 people had seen the video. So
\[
f(3) = \frac{500000}{1 + 4999e^{-3c}} = 500.
\]
Multiplying by $1 + 4999e^{-3c}$ and dividing by 500 gives
\[
1000 = 1 + 4999e^{-3c}.
\]
Subtracting 1 and dividing both sides by 4999 gives

\[
\frac{999}{4999} = e^{-3c}.
\]

We take \( \ln \) of both sides to get

\[-1.6102 = -3c,
\]

which means that

\[c = .537.
\]

So

\[f(t) = \frac{500000}{1 + 4999e^{-(.537)t}}.
\]

Finally, to find the number of people who have seen the video after 10 hours, plug in 10 for \( t \):

\[f(t) = \frac{500000}{1 + 4999e^{-(.537)10}} \approx 20605.
\]

9.) Adjusting for inflation, we use the formula

\[A = Pe^{rt}.
\]

In this case, to find out how much the 100 is worth in five years (with \( r = .05 \)), we have

\[A = 100e^{(.05)5} = 128.
\]

To find out how much the 150 is worth now, we plug in \( t = -5 \) (going 5 years backwards), giving

\[A = 150e^{(.05)(-5)} = 116.82.
\]

By both computations, the 150 in five years is worth more than the 100 now.

10.) We have 100 now and 150 in five years. So \( P = 100, A = 150, \) and \( t = 5. \) Plugging in,

\[150 = 100e^{5r}.
\]

Dividing by 100 gives

\[1.5 = e^{5r}.
\]

Taking \( \ln \) of both sides:

\[.405 = 5r,
\]

or

\[r = .081.
\]

So the inflation rate would have to be about 8.1%.

11.) To find this, we first split up the jackpot into 20 equal payments: each
payment will be \( \frac{184,000,000}{20} = 9,200,000 \) dollars. Now, the first payment, which is made now, is worth 9,200,000. The second payment, which is made in one year, is worth

\[
A = 9,200,000e^{-0.03}(1) = 8,928,099.09.
\]

The third payment, made in two years, will be

\[
A = 9,200,000e^{-0.03}(2) = 8,664,234.06.
\]

The fourth, made in three years, is

\[
A = 9,200,000e^{-0.03}(2) = 8,408,167.41.
\]

We calculate the fifth, sixth, seventh, and so on up to the twentieth (which is found by plugging in \( t = 19 \)). Adding these up gives

\[
Total = 9,200,000 + 8,928,099.09 + 8,664,234.06 + 8,408,167.41 + 8,159,668.68
+ 7,918,514.18 + 7,684,486.87 + 7,457,376.11 + 7,236,977.49 + 7,023,092.62
+ 6,815,529 + 6,614,999.81 + 6,418,623.75 + 6,228,924.88 + 6,044,832.45
+ 5,866,180.77 + 5,692,809.04 + 5,524,561.22 + 5,361,285.87 + 5202836.03
= 140,450,299.3
\]