Johns Hopkins Junior Number Theory Days 2020
December 4–5, 2020

SCHEDULE

All talks are 45′ long.
Please register at `https://bit.ly/3fxPi09` to receive the relevant links.

Friday 4 December

08:30	Welcome gathering for PhD students and post-docs.
09:00	**Shiva Chidambaram:** *Mod p Galois representations and abelian varieties.*
10:00	**Debanjana Kundu:** *Control theorems for fine Selmer groups..*
11:00	**Andrew Graham:** *Anticyclotomic Euler systems for conjugate self-dual representations of $GL(2n)$.*
12:00	**Razan Taha:** *P-adic measures for reciprocals of L-functions of totally real fields.*
12:45	Lunch break.
14:00	**Jize Yu:** *A geometric Jacquet–Langlands transfer via geometric Satake equivalence.*
15:00	**Zhilin Luo:** *A local trace formula for the local Gan-Gross-Prasad conjecture for special orthogonal groups.*
16:00	**Biao Wang:** *Analogues of Alladi's formula over global function fields.*
17:00	**Rahul Dalal:** *Statistics of automorphic representations through the stable trace formula.*

Saturday 5 December

08:30	Welcome gathering for PhD students and post-docs.
09:00	**Francesca Bianchi:** *P-adic heights and p-adic sigma functions on Jacobians of genus 2 curves.*
10:00	**Caleb Springer:** *Abelian varieties and their endomorphism rings.*
11:00	**Xiyuan Wang:** *The Tate conjecture for a concrete family of elliptic surfaces.*
12:00	**Soumya Sankar:** *Counting elliptic curves with a rational N-isogeny.*
12:45	Lunch break.
14:00	**Samuel Mundy:** *The Skinner–Urban method and the symmetric cube Bloch–Kato conjecture.*
15:00	**Congling Qiu:** *The Gross–Zagier–Zhang formula over function fields.*
16:00	**Josh Lam:** *Calabi–Yau varieties and Shimura varieties.*
17.00	**Kolya Malkin:** *Motivic fundamental groups of CM elliptic curves and geometry of Bianchi hyperbolic threefolds.*

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ABSTRACTS

Speaker: **Francesca Bianchi (Groningen).**
Title: *P-adic heights and p-adic sigma functions on Jacobians of genus 2 curves.*
Abstract: Extending work of Mazur and Tate on elliptic curves, Blakestad recently constructed a p-adic analogue of the complex sigma function on Jacobians of genus 2 curves. We use Blakestad's function to define, compute and study p-adic heights on such Jacobians. P-adic heights are of much arithmetic interest. Just to cite one application, they figure prominently in the quadratic Chabauty method for the computation of integral and rational points on curves.

Speaker: **Shiva Chidambaram (Chicago).**
Title: *Mod p Galois representations and abelian varieties.*
Abstract: For any Galois representation valued in $\mathrm{GSp}(2g, \mathbb{F}_p)$, one can ask whether it arises from the p-torsion of a g-dimensional abelian variety. A necessary condition, coming from Weil pairing, is that the similitude character of the representation be equal to the mod p cyclotomic character. This turns out to be sufficient for $(g, p) = (1, 2), (1, 3), (1, 5), (2, 2), (2, 3)$ and $(3, 2)$. In these cases, we discuss the question of parametrizing all abelian varieties with a fixed p-torsion representation, focusing on $(g, p) = (2, 3)$. When (g, p) is not one of these six tuples, we discuss how to make use of a local obstruction to produce representations that do not arise as torsion.

Speaker: **Rahul Dalal (Berkeley).**
Title: *Statistics of automorphic representations through the stable trace formula.*
Abstract: Since Automorphic representations for general groups are very difficult to study individually, they are often studied in families instead. The Arthur–Selberg trace formula lends itself very naturally to answering questions about averages of various parameters of the local components of automorphic representations in so-called harmonic families. In their 2016 work, Shin and Templier realized that the trace formula simplified dramatically enough in the special case of representations with discrete series at infinity that statistics were computable with good error bounds.

Shin–Templier's work used the invariant trace formula which disallowed families that distinguish representations with infinite component in the same L-packet. However, which member of an L-packet a representation might correspond to determines some important characteristicswhether the representation is holomorphic or quaternionic for example. I will discuss how methods related to the stable trace formula can remove this restriction. The key idea is applying a certain "hyperendoscopy" formulation of stabilization used first by Ferrari, though many technical difficulties come up.

Speaker: **Andrew Graham (Imperial College London–LSGNT).**
Title: *Anticyclotomic Euler systems for conjugate self-dual representations of $GL(2n)$.*
Abstract: An Euler system is a collection of Galois cohomology classes which satisfy certain compatibility relations under corestriction, and by constructing an Euler system and relating the classes to L-values, one can establish instances of the Bloch–Kato conjecture. In this talk, I will describe a construction of an anticyclotomic Euler system for a certain class of conjugate self-dual automorphic representations, which can be seen as a generalisation of the Heegner point construction. The classes arise from special cycles on unitary Shimura varieties and are closely related to the branching law associated with the spherical pair $(\mathrm{GL}(n) \times \mathrm{GL}(n), \mathrm{GL}(2n))$. This is joint work with S.W.A. Shah.

Speaker: **Debanjana Kundu (CRM Montréal).**
Title: *Control theorems for fine Selmer groups.*

Abstract: Inspired by the work of Iwasawa on growth of class groups in $\mathbb{Z}_p$-extensions, Mazur developed an analogous theory to study the growth of Selmer groups of Abelian varieties in $\mathbb{Z}_p$ -extensions. He proved a control theorem, which we describe briefly. Let A be an Abelian variety defined over a number field F with potential good ordinary reduction at all primes above p, and let $\mathcal{L}$ be a $\mathbb{Z}_p$ -extension of F. For every intermediate sub-extension F' of $\mathcal{L}/F$, we have natural maps

$$s_{\mathcal{L}/F'} : Sel(A/F') \to Sel(A/\mathcal{L})^{\mathrm{Gal}(\mathcal{L}/\mathrm{F}')}$$

on the Selmer groups, which are induced by the restriction maps on cohomology. Mazurs Control Theorem therefore asserts that the kernel and cokernel of the maps $s_{\mathcal{L}/F'}$ are finite and bounded independently of F'. This control theorem has subsequently been generalized to more general p-adic Lie extensions by Greenberg. We will consider variants of the control theorem for a certain subgroup of the p-primary Selmer group, called the fine Selmer group, which in recent years has been studied extensively. The said fine Selmer group is obtained by imposing stronger conditions at primes above p. We prove various Control Theorems for fine Selmer groups of elliptic curves in a general p-adic Lie extension, where the reduction type of the elliptic curve at primes above p may not be potentially good ordinary. We will discuss certain estimates on the $\mathbb{Z}_p$ -coranks of the kernel and cokernel of the restriction maps

$$r_{\mathcal{L}/F'} : R(E/F') \to R(E/\mathcal{L})^{\mathrm{Gal}(\mathcal{L}/\mathrm{F}')}$$

for a p-adic Lie extension $\mathcal{L}/F$. We will specialize to three cases of p-adic Lie extensions where we can show that the kernel and cokernel of the restriction map are finite, and (under appropriate assumptions) give growth estimates for their orders. This is joint work with Meng Fai Lim.

Speaker: **Josh Lam (Harvard).**
Title: *Calabi–Yau varieties and Shimura varieties.*
Abstract: Calabi–Yau (CY) varieties are certain special varieties which have been the subject of intense studies by algebraic geometers in the last few decades. I will try to explain some arithmetic properties of these varieties; more specifically, I will discuss two results on the Attractor Conjecture which was formulated by Greg Moore in 1998. Throughout I will emphasize the difference between CYs with and without Shimura moduli. Time permitting, I will discuss what one can (conjecturally!) expect from CYs with and without Shimura moduli. I will not assume familiarity with CYs or Shimura varieties; part of this is joint work with Arnav Tripathy.

Speaker: **Zhilin Luo (Minnesota).**
Title: *A local trace formula for the local Gan-Gross-Prasad conjecture for special orthogonal groups.*
Abstract: The local Gan–Gross–Prasad conjecture studies the restriction and branching problems for representations of classical and metaplectic groups. In this talk, I will talk about my proof for the tempered part of the local Gan–Gross–Prasad conjecture (multiplicity one in Vogan packets) for special orthogonal groups over any local fields of characteristic zero, which combines the work of Waldspurger (for the tempered part of the conjecture for special orthogonal groups over p-adic fields) and Beuzart-Plessis (for the tempered part of the conjecture for unitary groups over real field) in a non-trivial way. In the proof, an indispensable result which is also of independent interest is a formula expressing the regular nilpotent germs of quasi-split reductive Lie algebras over any local fields of characteristic zero via endoscopic invariants, which was previously proved by Shelstad over p-adic fields. We also relate the formula with the Kostant section.

Speaker: **Kolya Malkin (Yale).**
Title: *Motivic fundamental groups of CM elliptic curves and geometry of Bianchi hyperbolic threefolds.*
Abstract: I will describe a new construction related to the action of the motivic Galois group on the motivic fundamental groups of CM elliptic curves punctured at the p-torsion points. The l-adic realization of this action is the action of the absolute Galois group on the pro-l fundamental group.

The image of this action in the fundamental Lie algebra carries two canonical filtrations, by weight and depth. The depth-1 associated graded quotient was fully described by Beilinson and Levin using elliptic polylogarithms (in the Hodge realization). We describe it in depth 2, in the case when E is the Gaussian or Eisenstein CM elliptic curve. We construct a homomorphism from a chain complex computing cohomology of the Bianchi orbifold $\Gamma_1(p)\backslash\mathcal{H}_3$ to the standard cochain complex of the image of the above action.

This work generalizes several results of A. Goncharov and of the author on the motivic fundamental group of $\mathbb{G}_m$ punctured at roots of unity and its connection with modular manifolds. Reference: `https://arxiv.org/pdf/2010.07238.pdf`.

Speaker: **Samuel Mundy (Columbia).**
Title: *The Skinner–Urban method and the symmetric cube Bloch–Kato conjecture.*
Abstract: I will discuss the Skinner–Urban method, which aims to construct nontrivial classes in certain Bloch–Kato Selmer groups, and explain recent work (in progress) carrying it out for symmetric cube Galois representations.

Speaker: **Congling Qiu (Yale).**
Title: *The Gross–Zagier–Zhang formula over function fields.*
Abstract: The Gross–Zagier formula relates heights of Heegner points on elliptic curves and derivatives of L-functions. This formula was generalized by Shouwu Zhang and Yuan–Zhang–Zhang. Over function fields, we prove such a formula in the format of Yuan–Zhang– Zhangs work, using arithmetic relative trace formulas.

Speaker: **Soumya Sankar (MSRI).**
Title: *Counting elliptic curves with a rational N-isogeny.*
Abstract: The classical problem of counting elliptic curves with a rational N-isogeny can be phrased in terms of counting rational points on certain moduli stacks of elliptic curves. Counting points on stacks poses various challenges, and I will discuss these along with a few ways to overcome them. In particular, I will answer the following question for certain values of N: how many elliptic curves of bounded naive height have a rational N-isogeny? The talk assumes no prior knowledge of stacks and is based on joint work with Brandon Boggess.

Speaker: **Caleb Springer (Penn State).**
Title: *Abelian varieties and their endomorphism rings.*
Abstract: The endomorphism ring of an elliptic curve defined over a finite field is an important and widely-studied object which is useful in many contexts, including cryptography, the investigation of isogeny graphs, and the determination of the structure of the group of rational points. In this talk, I will outline some generalizations of these results to abelian varieties of higher dimension, and present a subexponential algorithm for computing the endomorphism ring of an ordinary abelian surface defined over a finite field under certain technical assumptions. The heart of the algorithm, generalizing the method of Bisson and Sutherland for elliptic curves, exploits the ideal class groups of orders in CM fields, with an application of class field theory.

Speaker: **Biao Wang (Buffalo).**
Title: *Analogues of Alladi's formula over global function fields.*
Abstract: In 1977, Alladi introduced a duality between prime factors of integers, from which along with the prime number theorem in arithmetic progressions he showed the following beautiful formula on the Möbius function $\mu(n)$: if $(\ell, k) = 1$, then

$$-\sum_{\substack{n\geq 2\\ p_{\min}(n)\equiv \ell \pmod k}} \frac{\mu(n)}{n} = \frac{1}{\varphi(k)},$$

where $p_{\min}(n)$ is the smallest prime factor of n and φ is the Euler's totient function. Recently, this formula has been generalized from the Chebotarev density to the natural density of sets of primes in works by Dawsey, Sweeting and Woo, Kural, McDonald and Sah. In this talk I will discuss work with Lian Duan and Shaoyun Yi that proves the analogues of these results over global function fields.

Speaker: **Xiyuan Wang (Johns Hopkins).**
Title: *The Tate conjecture for a concrete family of elliptic surfaces.*
Abstract: We prove the Tate conjecture for a concrete family of elliptic surfaces. This is joint work with Lian Duan. In this talk, I will begin with a general introduction to the Tate conjecture and the Fontaine–Mazur conjecture. Then I will focus on the Tate conjecture for a family of elliptic surfaces introduced by Geemen and Top, and try to explain the motivation and the idea behind the proof.

Speaker: **Razan Taha (Purdue).**
Title: *P-adic measures for reciprocals of L-functions of totally real fields.*
Abstract: We construct a p-adic measure whose Mellin transform interpolates the special values of the reciprocal of the p-adic L-function of a totally real field K. This measure is defined by analyzing the non-constant terms in the Fourier expansion of certain Eisenstein series of the Hilbert modular group $\mathrm{SL}(2, \mathcal{O})$.

Speaker: **Jize Yu (IAS).**
Title: *A geometric Jacquet–Langlands transfer via geometric Satake equivalence.*
Abstract: Geometric approaches in constructing the Jacquet–Langlands transfers were pioneered by Ribet and Serre. Their works motivate many of the later progress in this direction. In this talk, I will discuss a geometric construction of the Jacquet–Langlands transfer for automorphic forms of higher weights. The essential ingredient of the construction is the integral coefficient geometric Satake equivalence in mixed characteristic obtained by the speaker.

ORGANIZERS:
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