

Johns Hopkins Junior Number Theory Days 2019

December 7–8, 2019

SCHEDULE

Saturday 7 December

All lectures in Krieger Hall 205.

- 08:45 Breakfast in Krieger 413.
09:10 **Patrick Daniels:** *A Tannakian framework for displays and Rapoport–Zink spaces.*
09:55 Coffee break.
10:15 **María Inés de Frutos Fernández:** *Moduli Spaces of Shtukas over the Projective Line.*
11:10 **Alexander Bertoloni Meli:** *The Langlands Correspondence and Rapoport–Zink Spaces.*
12:00 Lunch break.
13:30 **Mathilde Gerbelli-Gauthier:** *Cohomology of Arithmetic Groups and Endoscopy.*
14:25 **Liyang Yang:** *Analytic Continuation of Certain L-functions.*
15:10 Coffee break.
15:40 **Bin Guan:** *Averages of central values of triple product L-functions.*
16:35 **Levent Alpoge:** *TBA.*
18:30 Dinner .
Non-speakers: please register at <http://math.jhu.edu/~sakellar/JNTD2019/registration.html> for dinner.

Sunday 8 December

All lectures in Krieger Hall 205.

- 08:45 Breakfast in Krieger 413.
09:10 **Chi-Yun Hsu:** *Construction of Euler Systems for $GSp_4 \times GL_2$.*
09:55 Coffee break.
10:15 **John Enns:** *Mod p local-global compatibility for $GSp_4(\mathbb{Q}_p)$ and the Jantzen filtration.*
11:10 **Hao Li:** *The congruence relation for $GSpin$ Shimura varieties.*
12:00 Lunch break.
13:30 **Wanlin Li:** *Newton Polygon Stratification of the Torelli Locus in PEL-type Shimura Varieties.*
14:25 **Huy Dang:** *Hurwitz tree and equal characteristic deformation of Artin-Schreier covers.*
15:10 Coffee break.
15:40 **Atticus Christensen:** *Specialization of Neron–Severi groups in characteristic $p > 0$.*

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ABSTRACTS

Speaker: **Levent Alpoge (Princeton).**

Title: *TBA.*

Abstract: TBA

Speaker: **Alexander Bertoloni Meli (Berkeley).**

Title: *The Langlands Correspondence and Rapoport–Zink Spaces.*

Abstract: The connection between the cohomology of Rapoport–Zink spaces and the local Langlands correspondence has been considered by many authors. In particular, a conjecture of Kottwitz describes the supercuspidal part of the cohomology in terms of the Langlands correspondence. In this talk, we show how to extend this conjecture in the general linear case to more general representations and give an indication as to how to approach this question for other groups.

Speaker: **Atticus Christensen (MIT).**

Title: *Specialization of Neron–Severi groups in characteristic $p > 0$.*

Abstract: We will present two results on the specialization of Neron–Severi groups in characteristic $p > 0$. The first bounds the exponent of p which kills the cokernel of specialization of Neron–Severi groups coming from any family of algebraic varieties. The second result is analogous to a result by Maulik and Poonen in mixed characteristic and establishes that given a smooth projective family of algebraic varieties over an algebraically closed field K of characteristic $p > 0$ not isomorphic to $\overline{\mathbb{F}}_p$, there exists a K point outside of the Noether-Lefschetz locus. We will explain how Morrow’s Lefschetz (1, 1) for crystalline cohomology is used to prove the first result and how the second result follows from the first.

Speaker: **Patrick Daniels (Maryland).**

Title: *A Tannakian framework for displays and Rapoport–Zink spaces.*

Abstract: We develop a Tannakian framework for group-theoretic analogs of displays, originally introduced by Bueltel and Pappas, and further studied by Lau. We use this framework to generalize the purely group-theoretic definition of Rapoport–Zink spaces given by Bueltel and Pappas, and to show that this definition coincides with the classical one in the case of unramified EL-type local Shimura data.

Speaker: **Huy Dang (Virginia).**

Title: *Hurwitz tree and equal characteristic deformation of Artin-Schreier covers.*

Abstract: An Artin–Schreier curve is a \mathbf{Z}/p -branched cover of the projective line over a field of characteristic $p > 0$. A unique aspect of positive characteristic is that there exist flat deformations of a wildly ramified cover that change the number of branch points but fix the genus. In this talk, we introduce the notion of Hurwitz tree. It is a combinatorial-differential object that is endowed with essential degeneration data of a deformation. We then show how the existence of a deformation between two covers with different branching data equates to the existence of a Hurwitz tree with behaviors determined by the branching data. One application of this result is to prove that the moduli space of Artin–Schreier covers of fixed genus g is connected when g is sufficiently large. If time permits, we will describe a generalization of the technique for all cyclic covers and the lifting problem.

Speaker: **John Enns (Northwestern).**

Title: *Mod p local-global compatibility for $\mathrm{GSp}_4(\mathbb{Q}_p)$ and the Jantzen filtration.*

Abstract: Given a Galois representation $\bar{\rho} : G_{\mathbb{Q}_p} \rightarrow \mathrm{GL}_n(\overline{\mathbb{F}}_p)$, a global construction of Caraiani-Emerton-Gee-Geraghty-Paškūnas-Shin furnishes a smooth $\overline{\mathbb{F}}_p$ -representation Π of $\mathrm{GL}_n(\mathbb{Q}_p)$ that one hopes realizes a mod p local Langlands correspondence. It is not obvious that this construction doesn't forget any information about $\bar{\rho}$. I will explain how the data of $\bar{\rho}$ gets encoded in this smooth representation in certain cases by connecting the deformation theory of $\bar{\rho}$ with data arising from Π via Jantzen's filtration of principal series representations for finite Chevalley groups. This allows me to prove new results in an analogous case for $\mathrm{GSp}_4(\mathbb{Q}_p)$.

Speaker: **María Inés de Frutos Fernández (Boston University)**.

Title: *Moduli Spaces of Shtukas over the Projective Line.*

Abstract: There are two notions of modularity of an elliptic curve E defined over a function field: analytic modularity (the L -function of E agrees with that of an automorphic form), and geometric modularity (under mild conditions on the conductor of E , there is a uniformization from a Drinfeld modular curve). We will describe a conjectured geometric modularity statement involving moduli spaces of Drinfeld shtukas, along with some explicit descriptions of moduli spaces of shtukas over the projective line which might help to check certain cases of the modularity statement.

Speaker: **Mathilde Gerbelli-Gauthier (Chicago)**.

Title: *Cohomology of Arithmetic Groups and Endoscopy.*

Abstract: I will discuss the problem of computing asymptotics of Betti numbers of congruence subgroups in unitary groups as a function of the level. In degrees below the middle, these dimensions of cohomology grow sub-linearly in the volume of the corresponding locally symmetric space. I will present a strategy to compute the exact growth using automorphic representations and the stable trace formula.

Speaker: **Bin Guan (CUNY)**.

Title: *Averages of central values of triple product L -functions.*

Abstract: Feigon and Whitehouse studied central values of triple L -functions averaged over newforms of weight 2 and prime level. They proved some exact formulas applying the results of Gross and Kudla which link central values of triple L -functions to classical "periods". In this talk, I will show more results of this problem for more cases using Jacquet's relative trace formula and Ichino's period formula, and some application of these average formulas to the non-vanishing problem.

Speaker: **Chi-Yun Hsu (UCLA)**.

Title: *Construction of Euler Systems for $\mathrm{GSp}_4 \times \mathrm{GL}_2$.*

Abstract: An Euler system is a collection of norm-compatible first Galois cohomology classes with the Galois groups varying over cyclotomic fields. By constructing an Euler system, one can bound the Selmer group of Galois representations. We construct Euler systems for the Galois representations coming from automorphic representations of $\mathrm{GSp}_4 \times \mathrm{GL}_2$. The strategy follows the work of Loeffler-Zerbes-Skinner in the case of GSp_4 , using automorphic input to show norm compatibility. This is a work in progress with Zhaorong Jin and Ryotaro Sakamoto.

Speaker: **Hao Li (Boston College)**.

Title: *The congruence relation for GSpin Shimura varieties.*

Abstract: Eichler-Shimura congruence relation relates the Hecke operator T_p and Frobenius action on its Jacobian (a kind of "(co)homology") mod p , for p a prime of good reduction. As a theorem relating the Galois world and the automorphic world, it played an important role in Wiles proof of modularity theorem. In their paper, Blasius and Rogawski reformulated the congruence relation in the framework of Langlands program and conjectured it for all Shimura varieties and Chai-Faltings gave a variant of this conjecture in their book. I will prove the C-F version of this conjecture in my talk for GSpin Shimura varieties, based on our recent better understanding of its integral model and special fiber. Time permitting, I will explain why the "fundamental expectation" on the cohomology of Shimura varieties predicts such a relation.

Speaker: **Wanlin Li (MIT)**.

Title: *Newton Polygon Stratification of the Torelli Locus in PEL-type Shimura Varieties*.

Abstract: A fundamental problem in arithmetic geometry is to determine which abelian varieties arise as Jacobians of (smooth) curves. In positive characteristic p , we study this problem from the moduli perspective by asking which Newton strata intersect the Torelli locus in the moduli of abelian varieties. In this talk, I will introduce a general picture where we try to answer his question by replacing A_g with a Shimura variety of PEL-type, and M_g with a Hurwitz space of cyclic covers of \mathbf{P}^1 . Using an inductive method, when $p \equiv 2 \pmod{3}$, for all g , we prove the existence of a smooth curve of genus g whose Newton polygon has about $2g/3$ slopes of $1/2$. This work is joint with Mantovan, Pries and Tang.

Speaker: **Liyang Yang (Caltech)**.

Title: *Analytic Continuation of Certain L -functions*.

Abstract: In this talk, we will mainly discuss two conjectures on entireness of certain basic L -functions from Galois representation and automorphic representation. Although the background and definitions are quite different, we will show these L -functions are closely related to each other by Jacquet–Zagier trace formula. As a consequence, we obtain holomorphy of adjoint L -functions for $\mathrm{GL}(n)$, where $n \leq 4$.

ORGANIZERS:

Yiannis Sakellaridis and David Savitt

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