

# Rutgers-Newark Junior Number Theory Days 2018

## November 2–3, 2018

### SCHEDULE

#### Friday 2 November

All lectures in Boyden Hall Room 413, 195 University Avenue, Newark.

- 08:45 Coffee and breakfast.  
09:00 Welcoming remarks.  
09:10 **Tony Feng**: A base change fundamental lemma via the geometry of shtukas.  
09:55 Coffee break.  
10:15 **Jingwei Xiao**: Endoscopic fundamental lemma for unitary groups and the Jacquet-Rallis transfer.  
11:10 **Maria Fox**: The  $GL_4$  Rapoport-Zink Space.  
12:00 Lunch break.  
14:00 **Lawrence Vu**: Hermitian Maass lift of general level.  
14:45 Coffee break.  
15:15 **Jun Su**: Automorphy of coherent cohomology of Shimura varieties.  
16:10 **Alex Carney**: The arithmetic Hodge-index theorem and dynamical systems.  
17:45 Meet at Hilton Newark Penn Station lobby.  
18:00 Dinner at Casa Vasca, 141 Elm Street, Newark.  
*Non-speakers: please register at <http://math.newark.rutgers.edu/~sakellar/JNTD2018/registration.html> for dinner.*

#### Saturday 3 November

All lectures in Boyden Hall Room 413, 195 University Avenue, Newark.

- 08:45 Coffee and breakfast.  
09:10 **Michele Fornea**: Hirzebruch–Zagier cycles and the BSD conjecture over quintic fields.  
09:55 Coffee break.  
10:15 **Daniel Kriz**: A new  $p$ -adic Maass–Shimura operator and supersingular Rankin–Selberg  $p$ -adic  $L$ -functions.  
11:10 **Nicholas Triantafillou**: The Method of Chabauty–Coleman–Skolem for restrictions of scalars.

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**ABSTRACTS**

**Speaker: Alex Carney (UC Berkeley).**

*Title: The arithmetic Hodge-index theorem and dynamical systems.*

**Abstract:** The Hodge-index theorem in classical algebraic geometry states that the signature of the intersection form on a surface is  $+1, -1, \dots, -1$ . In one of the fundamental results of Arakelov theory, Faltings and Hriljac extend this to arithmetic surfaces by relating the intersection pairing to the negative of the Néron–Tate height pairing. In this talk, I’ll explain how Yuan and Zhang (in the number field case) and my work (in the function field case) generalize this to higher dimensional varieties. An important use of arithmetic intersection theory is in defining height functions of both points and subvarieties, and as an example I will show how the Hodge-index theorem is used to prove a rigidity theorem for preperiodic points in algebraic dynamical systems.

**Speaker: Tony Feng (Stanford).**

*Title: A base change fundamental lemma via the geometry of shtukas.*

**Abstract:** We will explain a geometric approach to the base change fundamental lemma over function fields, which is based on ideas of Ngo Bao Chau. The approach works by comparing the cohomology groups of moduli spaces of shtukas, which we will introduce.

**Speaker: Michele Fornea (McGill University).**

*Title: Hirzebruch–Zagier cycles and the BSD conjecture over quintic fields.*

**Abstract:** We will report on a joint work in progress with Zhaorong Jin, where we give a geometric construction of twisted triple product  $p$ -adic  $L$ -functions when  $p$  splits in the relevant real quadratic field. As an application, we establish new instances of the rank zero BSD conjecture over quintic fields.

**Speaker: Maria Fox (Boston College).**

*Title: The  $GL_4$  Rapoport–Zink Space.*

**Abstract:** The  $GL_{2n}$  Rapoport–Zink space is a moduli space of supersingular  $p$ -divisible groups of dimension  $n$  and height  $2n$ , with a quasi-isogeny to a fixed base point. After the  $GL_2$  Rapoport–Zink space, which is zero-dimensional, the  $GL_4$  Rapoport–Zink space has the most fundamental moduli description, yet relatively little of its specific geometry has been explored. In this talk, I will give a full description of the geometry of the  $GL_4$  Rapoport–Zink space, including the connected components, irreducible components, and intersection behavior of the irreducible components. As an application of the main result, I will also give a description of the supersingular locus of the Shimura variety for the group  $GU(2, 2)$  over a prime split in the corresponding imaginary quadratic field.

**Speaker: Daniel Kriz (MIT).**

*Title: A new  $p$ -adic Maass–Shimura operator and supersingular Rankin–Selberg  $p$ -adic  $L$ -functions.*

**Abstract:** We introduce a new  $p$ -adic Maass–Shimura operator acting on a space of “generalized  $p$ -adic modular forms” (extending Katz’s notion of  $p$ -adic modular forms), defined on the  $p$ -adic (preperfectoid) universal cover of a Shimura curve. Using this operator, we construct new  $p$ -adic  $L$ -functions in the style of Katz, Bertolini–Darmon–Prasanna and Liu–Zhang–Zhang for Rankin–Selberg families over imaginary quadratic fields  $K$ , in the “supersingular” case where  $p$  is inert or ramified in  $K$ . We also establish new  $p$ -adic Waldspurger formulas, relating  $p$ -adic logarithms of elliptic units and Heegner points to special values of these  $p$ -adic  $L$ -functions. If time permits, we will discuss some applications to the arithmetic of abelian varieties.

Speaker: **Jun Su (Princeton).**

Title: *Automorphy of coherent cohomology of Shimura varieties.*

Abstract: What are holomorphic/nearly holomorphic automorphic forms in general? The rough answer is global sections of automorphic vector bundles over Shimura varieties, while to be precise for a non-compact Shimura variety one need to take a toroidal compactification of it and consider the canonical extensions of automorphic vector bundles over the compactification. Recently people (Emerton–Reduzzi–Xiao, Boxer/Goldring–Koskivirta, Pilloni–Stroh) have attached Galois representations to eigenclasses in the higher cohomology of these sheaves, and on the automorphic/Hecke side we prove that these higher cohomology can also be computed by automorphic forms. In this talk we will introduce the relevant constructions, motivate why those cohomology groups could be interesting, and shed some light on our proof.

Speaker: **Nicholas Triantafillou (MIT).**

Title: *The Method of Chabauty–Coleman–Skolem for Restrictions of Scalars.*

Abstract: For a number field  $K$ , a finite set  $S$  of primes of  $K$  and an arithmetic curve  $C$  over the  $S$ -integers of  $K$ , the method of Chabauty–Coleman–Skolem (Chabauty’s method) is a powerful tool for bounding/enumerating the set  $C(\mathcal{O}_{K,S})$ . The method typically requires that dimension of the generalized Jacobian  $J$  of  $C$  is greater than the rank of  $J(\mathcal{O}_{K,S})$ . Since this condition often fails, especially when  $[K : \mathbb{Q}]$  is large, several techniques have been proposed to augment Chabauty’s method. For proper curves, Siksek introduced an analogue of Chabauty’s method for the restriction of scalars  $\text{Res}_{K/\mathbb{Q}}C$  that can succeed when the rank of  $J(\mathcal{O}_{K,S})$  is as large as  $[K : \mathbb{Q}] \cdot (\dim J - 1)$ . We develop Chabauty’s method for restrictions of scalars in the non-proper case and study the power of this approach for computing  $C = (\mathbb{P}^1 \setminus \{0, 1, \infty\})(\mathcal{O}_{K,S})$ . As an application, we show that if 3 splits completely in  $K$  then there are no solutions to the unit equation  $x + y = 1$  with  $x, y \in \mathcal{O}_K^\times$ .

Speaker: **Lawrence Vu (CUNY Graduate Center).**

Title: *Hermitian Maass lift of general level.*

Abstract: The Maass lift, a lift of elliptic modular form to a Hermitian modular form (in other words, the Hermitian analogue of the well-known Saito–Kurokawa lift for Siegel modular form), has number theoretic interest such as lending evidences to the important Bloch–Kato conjecture. There has been numerous construction of such a lift for level 1; starting with the work of Kojima in 1982, followed by those of Gritsenko, Krieg, Ikeda, Atobe. In this talk, I shall describe my recent work on the generalization of to arbitrary level.

Speaker: **Jingwei Xiao (MIT).**

Title: *Endoscopic fundamental lemma for unitary groups and the Jacquet–Rallis transfer.*

Abstract: We give another proof of the endoscopic fundamental lemma (theorem of Laumon–Ngo) for unitary Lie algebras in characteristic 0. We study the compatibility between Fourier transform and transfer and we prove that the compatibility in Jacquet–Rallis setting, when taking limits using germ expansions, will imply the compatibility in the endoscopic setting for unitary group.

ORGANIZERS:

**Zhengyu Mao and Yiannis Sakellaridis**

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