Rutgers-Newark Junior Number Theory Days 2017
November 17–18, 2017

SCHEDULE

Friday 17 November

All lectures in Boyden Hall Room 100, 195 University Avenue, Newark.

09:00	Welcoming remarks.
09:10	**Junho Peter Whang**: Nonlinear descent on moduli of local systems.
09:55	Coffee break.
10:15	**Travis Morrison**: Reductions between supersingular elliptic curve isogeny and endomorphism ring problems.
11:10	**Zev Rosengarten**: Tamagawa Numbers of Linear Algebraic Groups.
12:00	Lunch break.
13:30	**Charlotte Chan**: Period identities of CM forms on quaternion algebras.
14:25	**Zheng Liu**: p-adic L-functions for ordinary families on symplectic groups.
15:10	Coffee break.
15:40	**Spencer Leslie**: A Generalized Theta lifting and CAP representations.
16:35	**Shenhui Liu**: Central L-values of $\mathrm{GL}(3)$ Maass forms.
18:15	Meet at Hilton Newark Penn Station lobby.
18:30	Dinner at Casa Vasca, 141 Elm Street, Newark. *Non-speakers: please register at* `http://math.newark.rutgers.edu/~sakellar/JNTD2017/registration.html` *for dinner.*

Saturday 19 November

All lectures in Boyden Hall Room 100, 195 University Avenue, Newark.

09:10	**Jingren Chi**: The geometry of some group analogue of affine Springer fibers.
09:55	Coffee break.
10:15	**Jonathan Wang**: The Drinfeld–Gaitsgory operator on automorphic functions.
11:10	**Renee Bell**: Local-to-Global Lifting to Curves in Characteristic p.
12:00	Lunch break.
13:30	**Koji Shimizu**: Around the relative Fontaine–Mazur conjecture.
14:25	**Lue Pan**: Fontaine–Mazur conjecture in the residually reducible case.
15:10	Coffee break.
15.40	**Rong Zhou**: The special fiber of Shimura varieties.

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ABSTRACTS

Speaker: **Junho Peter Whang (Princeton).**
Title: *Nonlinear descent on moduli of local systems.*
Abstract: In 1880, Markoff studied the cubic Diophantine equation $x^2 + y^2 + z^2 = 3xyz$ and observed that its integral solutions satisfy a form of descent under nonlinear polynomial transformations. Generalizing this, in this talk we consider families of moduli spaces for local systems on topological surfaces, and prove a structure theorem for their integral points using mapping class group dynamics. The result is reminiscent of the finiteness of class numbers for linear arithmetic group actions after Gauss, Minkowski, and others.

Speaker: **Travis Morrison, (Penn State).**
Title: *Reductions between supersingular elliptic curve isogeny and endomorphism ring problems.*
Abstract: Given a finite field k and an elliptic curve E/k, computing $\mathrm{End}(E)$ is a fundamental problem, as it is closely related to computing the k-rational points of E. Less is known about the complexity of computing $\mathrm{End}(E)$ when E is supersingular compared to the case when E is ordinary. This problem is now also a practical one: NIST is currently soliciting submissions of post-quantum cryptographic protocols, meaning cryptosystems which would be secure even against a quantum computer. Protocols whose security is based on the hardness of computing isogenies between supersingular elliptic curves have been suggested for hashes, key sharing, public key encryption, and signature schemes. Breaking these systems reduces to the problems of either computing an isogeny of prime-power degree between two given supersingular elliptic curves, or computing the endomorphism ring $\mathrm{End}(E)$ of a supersingular elliptic curve E (meaning compute a maximal order of a quaternion algebra isomorphic to $\mathrm{End}(E)$).

The isogeny and endomorphism ring problems are deeply related, and are often referred to as equivalent problems. However, there are no concrete, polynomial time reductions from one to the other. In joint work with Kirsten Eisentraeger and Sean Hallgren, we study the size of endomorphism rings and maximal orders, which is necessary to have a meaningful reduction which involves these objects. Additionally, we give a polynomial time reduction from the problem of computing an ℓ-power isogeny between two given supersingular curves for a prime ℓ to the problem of, given a supersingular curve E, computing both the maximal order isomorphic to $\mathrm{End}(E)$ along with the action of $\mathrm{End}(E)$ on the ℓ-torsion of E. Thus we reduce the problem of computing isogenies to a problem of computing endomorphism rings, meaning knowing the algebraic structure of $\mathrm{End}(E)$ along with a little bit of its geometric structure.

Speaker: **Zev Rosengarten (Stanford).**
Title: *Tamagawa Numbers of Linear Algebraic Groups.*
Abstract: In 1981, Sansuc obtained a formula for Tamagawa numbers of reductive groups over number fields, modulo some then unknown results on the arithmetic of simply-connected groups which have since been proven, particularly Weil's conjecture on Tamagawa numbers over number fields. One easily deduces that this same formula holds for all linear algebraic groups over number fields. Sansuc's method still works to treat reductive groups in the function field setting, thanks to the recent resolution of Weil's conjecture in the function field setting by Lurie and Gaitsgory. However, due to the imperfection of function fields, the reductive case is very far from the general one; indeed, Sansuc's formula does not hold for all linear algebraic groups over function fields. We propose a modification of Sansuc's formula that recaptures it in the number field case and also gives a correct answer over number fields. We have proven this formula for all pseudo-reductive groups in characteristic greater than 3, as well as for all commutative groups (in any characteristic). The commutative case (which is essential even for the general

pseudo-reductive case) is a corollary of a vast generalization of the Poitou-Tate nine-term exact sequence, from finite group schemes to arbitrary affine commutative group schemes of finite type.

Speaker: **Charlotte Chan (Michigan).**
Title: *Period identities of CM forms on quaternion algebras.*
Abstract: In groundbreaking work in 1985, Waldspurger proved an identity between the central value of an L-function and the norm of a torus period. By the Jacquet–Langlands correspondence, this implies that the norm of torus periods on different quaternion algebras are related. In this talk, we will show that for automorphic forms coming from Hecke characters of CM fields, there is an identity of the torus periods themselves.

Speaker: **Zheng Liu (IAS/McGill).**
Title: *p-adic L-functions for ordinary families on symplectic groups.*
Using the doubling method, we study the p-adic congruences among critical values of the standard L-functions attached to Siegel modular forms, and construct the p-adic L-functions for ordinary families of Hecke eigen-systems of the symplectic group $\mathrm{Sp}(2n)/\mathbb{Q}$. A key step in the construction is to choose suitable local sections for the Siegel Eisenstein series on $\mathrm{Sp}(4n)/\mathbb{Q}$. We explain the strategy for making the choices.

Speaker: **Spencer Leslie (Boston College).**
Title: *A Generalized Theta lifting and CAP representations.*
Abstract: We discuss a new lifting of automorphic representations using the generalized theta representation on the 4-fold cover of the symplectic group. A key feature is that this lift produces CAP representations, which are counterexamples of the generalized Ramanujan conjecture. This motivates a connection to the emerging "Langlands program for covering groups" by way of Arthur parameters. The crucial fact allowing this lift to work is that theta functions for the 4-fold cover still have few non-vanishing Fourier coefficients, which fails for higher-degree covers.

Speaker: **Shenhui Liu (Toronto).**
Title: *Central L-values of GL(3) Maass forms.*
Abstract: In this talk, we consider certain GL(3) L-functions at the central point of the critical strip. Specifically, consider an orthogonal basis $\{\phi_j\}$ of Hecke–Maass forms for $SL(3,\mathbb{Z})$. By the method of moments and the mollification method, we obtain a positive-proportional nonvanishing result for $L(\frac{1}{2},\phi_j)$ when the spectral parameters of ϕ_j are concentrated around a large parameter T. The main tool we use is the GL(3) Kuznetsov formula. This is joint work with Bingrong Huang and Zhao Xu.

Speaker: **Jingren Chi (Chicago).**
Title: *The geometry of some group analogue of affine Springer fibers.*
Abstract: The group analogue of affine Springer fibers (as opposed to the more classical Lie algebra affine Springer fibers) are certain algebraic varieties that arises naturally in the study of orbital integrals of spherical Hecke functions on a p-adic group. In this talk, I will discuss the current state of knowledge on basic geometric properties of these varieties. After reporting joint work with Alexis Bouthier on the dimension formula, I will formulate a conjecture which relates the number of irreducible components of these varieties to certain weight multiplicities defined by the Langlands dual group. The emphasis will be the on the differences from the classical Lie algebra situation.

Speaker: **Jonathan Wang (IAS).**
Title: *The Drinfeld–Gaitsgory operator on automorphic functions.*
Abstract: Let F be a function field and G a connected split reductive group over F. We define a "strange" operator between spaces of automorphic functions on G(A)/G(F), and show that this operator is natural from the viewpoint of the geometric Langlands program via the functions-sheaves dictionary. We discuss

how to define this operator over a number field by relating it to pseudo-Eisenstein series and inversion of the standard intertwining operator. The Drinfeld-Gaitsgory operator is also connected to Deligne-Lusztig duality and cohomological duality of representations over a local field.

Speaker: **Renee Bell (MIT).**
Title: *Local-to-Global Lifting to Curves in Characteristic p.*
Abstract: Given a Galois cover of curves $X \to Y$ with Galois group G which is totally ramified at a point x and unramified elsewhere, restriction to the punctured formal neighborhood of x induces a Galois extension of Laurent series rings $k((u))/k((t))$. If we fix a base curve Y, we can ask when a Galois extension of Laurent series rings comes from a global cover of Y in this way. Harbater proved that over a separably closed field, this local-to-global principle holds for any base curve if G is a p-group, and gave a condition for the uniqueness of such an extension. Using a generalization of Artin-Schreier theory to non-abelian p-groups, we characterize the curves Y for which this lifting property holds and when it is unique, but over a more general ground field.

Speaker: **Koji Shimizu (Harvard).**
Title: *Around the relative Fontaine–Mazur conjecture.*
Abstract: The l-adic étale cohomology of an algebraic variety over $\mathbb{Q}$ gives an example of l-adic Galois representations. Moreover, such a Galois representation is unramified almost everywhere and satisfies some local condition at l formulated in p-adic Hodge theory. Fontaine and Mazur conjectured that the converse also holds. My talk is about the relative version of this conjecture replacing the Galois group by the étale fundamental group of a variety. I will explain a result on the existence of a compatible system of an étale local system, which gives evidence of the relative Fontaine–Mazur conjecture.

Speaker: **Lue Pan (Princeton).**
Title: *Fontaine–Mazur conjecture in the residually reducible case.*
Abstract: We prove the modularity of some two-dimensional residually reducible Galois representations (of $G_{\mathbb{Q}}$) under certain hypothesis on the residual representation at p. To do this, we generalize Emerton's local-global compatibility and devise a patching argument for completed homology in this setting.

Speaker: **Rong Zhou (IAS).**
Title: *The special fiber of Shimura varieties.*
Abstract: The Langlands–Rapoport conjecture gives a description of the points in the special fiber of certain integral models for Shimura varieties. Results of this kind are useful in understanding things like the Hasse-Weil zeta function of the Shimura variety. In this talk we show, under certain restrictions on the group, that the isogeny classes in the special fiber of the integral models constructed by Kisin and Pappas have the form predicted by the conjecture and use this to deduce a result on the non-emptiness of Newton strata.

ORGANIZERS:
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