

Rutgers-Newark Junior Number Theory Days 2016

November 18-19, 2016

SCHEDULE

Friday 18 November

All lectures in Conklin Hall Room 100, 175 University Avenue, Newark.

- 09:00 Welcoming remarks.
- 09:10 **Padmavathi Srinivasan**: Conductors and minimal discriminants of hyperelliptic curves with rational Weierstrass points.
- 09:55 Coffee break.
- 10:15 **Anastassia Etropolski**: Sporadic Torsion on Elliptic Curves.
- 11:10 **Wade Hindes**: Integral points in dynamical orbits.
- 12:00 Lunch break.
- 13:30 **Joel Specter**: The macroscopic properties of polynomials over finite fields.
- 14:25 **Brett Frankel**: \mathbb{F}_q -Local systems on abelian varieties of low p -rank.
- 15:10 Coffee break.
- 15:40 **Eugenia Rosu**: Integers that can be written as the sum of two cubes.
- 16:35 **Will Chen**: Moduli Interpretations for Noncongruence Modular Curves.
- 18:15 Meet at Hilton Newark Penn Station lobby.
- 18:30 Dinner at Casa Vasca, 141 Elm Street, Newark.
Non-speakers: please register here for dinner.

Saturday 19 November

All lectures in Boyden Hall Room 100, 195 University Avenue, Newark.

- 09:10 **Ryan Ronan**: An asymptotic for the growth of Markoff-Hurwitz tuples.
- 09:55 Coffee break.
- 10:15 **Alexandra Florea**: The mean value of quadratic Dirichlet L -functions in function fields.
- 11:10 **Liyang Zhang**: QUE of Degenerate Eisenstein series.
- 12:00 Lunch break.
- 13:30 **Yuanqing Cai**: Unique functionals on theta representations and Rankin-Selberg integrals for covering groups.
- 14:25 **Vladislav Petkov**: Cuspidal Theta representations.
- 15:10 Coffee break.
- 15:40 **Jon Cohen**: Transfer of representations and the Bernstein center for inner forms of $GL(n)$.
- 16:35 **Chen Wan**: Multiplicity one theorem for the Ginzburg-Rallis model.

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ABSTRACTS

Speaker: **Yuanqing Cai** (Boston College)

Title: *Unique functionals on theta representations and Rankin-Selberg integrals for covering groups.*

Abstract: It is difficult to obtain Euler products for covering groups since uniqueness of Whittaker models fails in this setting. In this talk, I'll describe some unique models on the theta representations and discuss its applications in Rankin-Selberg integrals for covering groups.

Speaker: **Will Chen** (I.A.S)

Title: *Moduli Interpretations for Noncongruence Modular Curves.*

Abstract: Let G be a finite 2-generated group. We will consider the moduli space of elliptic curves equipped with G -structures. When G is abelian, we recover the classical congruence level structures. When G is sufficiently non-abelian, the corresponding moduli spaces will be noncongruence modular curves. As time permits, we will describe connections with the Inverse Galois Problem and the Unbounded Denominators Conjecture for noncongruence modular forms.

Speaker: **Jon Cohen** (University of Maryland)

Title: *Transfer of representations and the Bernstein center for inner forms of $GL(n)$.*

Abstract: I will discuss a construction and characterization of the Local Langlands Correspondence for Inner Forms of GL_n , and its relation to a method for constructing explicit matching functions via a transference of Bernstein centers.

Speaker: **Anastassia Etropolski** (Rice)

Title: *Sporadic Torsion on Elliptic Curves.*

Abstract: In Mazur's celebrated 1978 Inventiones paper, he classifies the torsion subgroups which can occur in the Mordell-Weil group of an elliptic curve over \mathbb{Q} . His result was extended to elliptic curves over quadratic number fields by Kamienny, Kenku, and Momose, with the full classification being completed in 1992. What both of these cases have in common is that each subgroup in the classification occurs for infinitely many elliptic curves, but this no longer holds for cubic number fields. In 2012, Najman showed that there exists an elliptic curve whose torsion subgroup over a particular cubic field is $\mathbb{Z}/21$. This was the first "sporadic" example, and there is a very precise way to understand and classify such examples using the arithmetic of modular curves. I will show that this completes the classification of cubic torsion, as well as discuss how one should think of these sporadic examples and how one might search for others. This project is joint work with David Zureick-Brown and Jackson Morrow.

Speaker: **Alexandra Florea** (Stanford)

Title: *The mean value of quadratic Dirichlet L -functions in function fields.*

Abstract: I will talk about the first moment in the family of quadratic Dirichlet L -functions over function fields. Summing $L(1/2, \chi_D)$ over monic, square-free polynomials D of degree $2g + 1$ (with q fixed and $g \rightarrow \infty$), Andrade and Keating obtained an asymptotic formula with a main term of size $|D| \log_q |D|$. We will describe a different approach which allows us to go beyond the square-root cancellation from the corresponding number field problem. In particular, I will explain how to explicitly compute a secondary term of size $|D|^{1/3} \log_q |D|$.

Speaker: **Brett Frankel** (Northwestern)

Title: *\mathbb{F}_q -Local systems on abelian varieties of low p -rank.*

Abstract: For an abelian variety A with small p -torsion, we count the number of representations of the étale fundamental group of A to $\mathrm{GL}_n(q)$, where q is a power of p . This count (for fixed n) turns out to be a polynomial in q . The space of such representations is not a scheme, but does have the structure of a constructible set. We give an explicit formula for this polynomial, then state a few theorems which elucidate its features. In particular, we state a new result which generalizes to cosets a theorem of Frobenius about the number of solutions to $x^n = 1$ in a finite group.

Speaker: **Wade Hindes** (CUNY)

Title: *Integral points in dynamical orbits.*

Abstract: Over a number field K , a celebrated result of Silverman states that if $\varphi(z) \in K(z)$ is a rational function whose second iterate is not a polynomial, the set of integral points in the orbit $\mathcal{O}_\phi(P) = \{\phi^n(P)\}_{n \geq 0}$ is finite for all $P \in \mathbb{P}^1(K)$. In this talk, I will discuss Silverman's finiteness theorem from several new perspectives, including an "average" version and a generalization to global fields of finite characteristic.

Speaker: **Vladislav Petkov** (Columbia)

Title: *Cuspidal Theta representations.*

Abstract: The study of metaplectic covers of classical Lie groups started with the investigation of classical modular forms of half-integral weight. The Jacobi theta function $\theta(z)$, which is the inverse Mellin transform of the Riemann zeta function, is one of the famous examples of a half-integral weight form. A *theta representation* of a metaplectic group is an automorphic representation, which generalizes examples like $\theta(z)$. Kazhdan and Patterson construct such representations as the residues of metaplectic Eisenstein series on the degree n metaplectic cover of the general linear group GL_n . What makes these representations interesting is that they have a unique local Whittaker functional at every place. Gelbart and Piatetski-Shapiro called automorphic representations with this property distinguished, because for metaplectic representations the dimension of the local Whittaker models may be greater than one. Although every theta representation constructed by the global method of Eisenstein series is non-cuspidal, in rank 1 and 2 there exist cuspidal distinguished metaplectic representations. In this talk I will describe the construction of a family of cuspidal theta representations for GL_4 and discuss some possible generalizations.

Speaker: **Ryan Ronan** (CUNY)

Title: *An asymptotic for the growth of Markoff-Hurwitz tuples.*

Abstract: For integer parameters $n \geq 3$, $a \geq 1$, and $k \geq 0$ the Markoff-Hurwitz equation is the diophantine equation

$$x_1^2 + x_2^2 + \cdots + x_n^2 = ax_1x_2 \cdots x_n + k.$$

In this talk, we establish an asymptotic count for the number of integral solutions with $\max\{x_1, x_2, \dots, x_n\} \leq R$. When $n = a = 3$ and $k = 0$ this equation is known simply as the Markoff equation, for which the asymptotic count was studied in detail by Zagier in 1982. The previous best result for $n \geq 4$ is due to Baragar in 1998 who established an exponential rate of growth with exponent $\beta(n) > 0$ when $k = 0$. Surprisingly, this exponent β is not, in general, an integer. We use methods from symbolic dynamics to obtain a true asymptotic formula, and which yield a new interpretation of this exponent β in terms of the unique parameter for which there exists a certain conformal measure on projective space.

Joint work with Alex Gamburd and Michael Magee.

Speaker: **Eugenia Rosu** (U.C. Berkeley)

Title: *Integers that can be written as the sum of two cubes.*

Abstract: The Birch and Swinnerton-Dyer conjecture predicts that we have non-torsion rational points on an elliptic curve iff the L -function corresponding to the elliptic curve vanishes at 1. Thus BSD predicts that a positive integer N is the sum of two cubes if $L(E_N, 1) = 0$, where $L(E_N, s)$ is the L -function corresponding to the elliptic curve $E_N : x^3 + y^3 = N$. We have computed several formulas that relate $L(E_N, 1)$ to the trace of a modular function at a CM point. This offers a criterion for when the integer

N is the sum of two cubes. Furthermore, when $L(E_N, 1)$ is nonzero we get a formula for the number of elements in the Tate-Shafarevich group.

Speaker: **Joel Specter** (Northwestern)

Title: *The macroscopic properties of polynomials over finite fields.*

Abstract: The space of monic square-free polynomials over the complex numbers has a physical interpretation; it is the space of configurations of unordered particles in the plane. In physics, one uses statistical mechanics to explain macroscopic properties of a system in terms of microscopic ones. Over finite fields, I will show that certain polynomial statistics are similarly governed - they are determined by their statistics in small degrees. The reason for this is more than superficial, in this talk, I will discuss how the étale cohomology of the varieties parameterizing certain statistics carry a ring structure which arises from the little disk operad. As an application, I will give exact formulae for the distribution of polynomial discriminants over finite fields.

Speaker: **Padmavathi Srinivasan** (Georgia Tech)

Title: *Conductors and minimal discriminants of hyperelliptic curves with rational Weierstrass points.*

Abstract: Conductors and minimal discriminants are two measures of degeneracy of the singular fiber in a family of hyperelliptic curves. In the case of elliptic curves, the Ogg-Saito formula shows that (the negative of) the Artin conductor equals the minimal discriminant. In the case of genus two curves, equality no longer holds in general, but the two invariants are related by an inequality. We investigate the relation between these two invariants for hyperelliptic curves of arbitrary genus.

Speaker: **Chen Wan** (University of Minnesota)

Title: *Multiplicity one theorem for the Ginzburg-Rallis model.*

Abstract: Following the method developed by Waldspurger and Beuzart-Plessis in their proof of the local Gan-Gross-Prasad conjecture, we were able to prove the multiplicity one theorem on Vogan L -packet for the Ginzburg-Rallis model. In some cases, we also proved the epsilon dichotomy conjecture which gives a relation between the multiplicity and the value of the exterior cube epsilon factor.

Speaker: **Liyang Zhang** (Yale)

Title: *QUE of Degenerate Eisenstein series.*

Abstract: Arithmetic quantum unique ergodicity of Eisenstein series on $GL(2)$ was first formulated and proved by Luo and Sarnak. In this talk, we study the analog of Luo and Sarnaks result on a subspace of the $GL(n)$ continuous spectrum spanned by the degenerate Eisenstein series induced from the maximal parabolic subgroup with the constant function.

ORGANIZERS:

Zhengyu Mao and Yiannis Sakellaridis

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