

Rutgers-Newark Junior Number Theory Days 2015

November 20-21, 2015

SCHEDULE

All lectures in Conklin Hall Room 100, 175 University Avenue, Newark.

Friday 20 November

- 08:50 Welcoming remarks.
- 09:00 **Jessica Fintzen**: Stable vectors in the Moy-Prasad filtration
- 09:50 Coffee break.
- 10:20 **Beth Romano**: The Local Langlands Correspondence: New Examples from the Epipelagic Zone.
- 11:20 **Baiying Liu**: On discrete spectrum of quasi-split classical groups and the generalized Ramanujan problem.
- 12:10 Lunch break.
- 13:40 **Maria Nastasescu**: Non-vanishing of twists of L -functions of $GL(n)$ and applications.
- 14:30 Coffee break.
- 15:00 **Naser Talebizadeh Sardari**: Optimal strong approximation for quadratic forms.
- 16:00 **João Guerreiro**: Applications of the $GL(3)$ Kuznetsov trace formula.
- 17:00 Coffee break.
- 17:30 Public lecture by **Michael Harris**: Mathematics without apologies: Portrait of a problematic vocation.
- 18:45 Departure of shuttle & cars for the restaurant.
- 19:15 Dinner at Casa Vasca, 141 Elm Street, Newark.

Saturday 21 November

- 09:00 **Jason Polák**: Beginnings of relative endoscopy.
- 09:50 Coffee break.
- 10:20 **Ioan Filip**: A local relative trace formula for spherical varieties and distinguished representations.
- 11:20 **Cheng-Chiang Tsai**: Some representation theoretic objects in number theory.
- 12:10 Lunch break.
- 13:40 **Evangelia Gazaki**: On a filtration of CH_0 for an abelian variety.
- 14:40 **Jeremy Booher**: Geometric Deformations of Orthogonal and Symplectic Galois Representations.
- 15:30 Coffee break.
- 16:00 **Ari Shnidman**: Selmer groups in families of cubic twists.

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ABSTRACTS

Speaker: **Jeremy Booher** (Stanford)

Title: *Geometric Deformations of Orthogonal and Symplectic Galois Representations.*

Abstract: For a representation of the absolute Galois group of the rationals over a finite field of characteristic p , we would like to know if there exists a lift to characteristic zero with nice properties. In particular, we would like it to be geometric in the sense of the Fontaine-Mazur conjecture: ramified at finitely many primes and potentially semistable at p . For two-dimensional representations, Ramakrishna proved that under technical assumptions, odd representations admit geometric lifts. We generalize this to higher dimensional orthogonal and symplectic representations. The key ingredient is a smooth local deformation condition obtained by analysing unipotent orbits and their centralizers in the relative situation, not just over fields.

Speaker: **Ioan Filip** (Columbia University)

Title: *A local relative trace formula for spherical varieties and distinguished representations.*

Abstract: We derive the Plancherel formula for the space $X = \mathrm{GL}(2, F) \backslash \mathrm{GL}(2, E)$, with E/F a quadratic extension of local fields. We then describe a local trace formula in the setting of spherical varieties and focus on small rank cases.

Speaker: **Jessica Fintzen** (Harvard)

Title: *Stable vectors in the Moy-Prasad filtration*

Abstract: Reeder and Yu gave recently a new construction of certain supercuspidal representations of p -adic reductive groups (called epipelagic representations). Their construction relies on the existence of stable vectors in the first Moy-Prasad filtration quotient under the action of a reductive quotient. We will explain these ingredients and present a theorem about the existence of such stable vectors for all primes p . This builds on a result of Reeder and Yu about the existence of stable vectors for large primes.

Some of the above work forms part of a joint research project with Beth Romano.

Speaker: **Evangelia Gazaki** (University of Chicago)

Title: *On a filtration of CH_0 for an abelian variety.*

Abstract: Let A be an abelian variety over a field k of dimension d . In this talk I will define a decreasing filtration $\{F^r\}_{r \geq 0}$ of the group $CH_0(A)$ of zero cycles modulo rational equivalence which has the property that the successive quotients are "almost" isomorphic to some Milnor-type K -groups. Rationally the filtration coincides with the motivic filtration previously considered by A. Beauville and S. Bloch. We will see that when k is a p -adic field the filtration has many interesting properties. For example, we will obtain information on the kernel of the cycle map to étale cohomology, $CH_0(A)/n \rightarrow H^{2d}(A, \mu_n^{\otimes d})$, where n is a positive integer. If time permits, I will also present some work in progress, joint with K. Kato, regarding the image of the cycle map.

Speaker: **João Guerreiro** (Columbia University)

Title: *Applications of the $\mathrm{GL}(3)$ Kuznetsov trace formula.*

Abstract: The Kuznetsov trace formula on $\mathrm{SL}(3, Z)$ has yielded many number theoretic applications, such as results on the distribution of Maass forms and on symmetry types of families of L -functions. I will give an overview of this trace formula, focusing on the analysis of its geometric terms, and describe how one can obtain an orthogonality relation for the Fourier-Whittaker coefficients of a "thin" family of Maass forms.

Speaker: **Baiying Liu** (Princeton)

Title: *On discrete spectrum of quasi-split classical groups and the generalized Ramanujan problem.*

Abstract: Based on the theory of endoscopy, Arthur classified the automorphic discrete spectrum of quasi-split classical groups up to global Arthur packets parametrized by Arthur parameters. In this talk, I will address our recent progress on a few problems towards the refined structure of global Arthur packets. For example, a natural question one may ask is that whether a given Arthur packet has cuspidal representations or not, which has connection to the generalized Ramanujan problem: how bad the local components of the cuspidal spectrum could be?

Speaker: **Maria Nastasescu** (Caltech)

Title: *Non-vanishing of twists of L -functions of $GL(n)$ and applications.*

Abstract: Non-vanishing results for central values of automorphic L -functions have been of great interest in number theory. We present an application of the non-vanishing of twists of L -functions to the determination of the adjoint p -adic L -function of an elliptic curve. More specifically, we fix p an odd prime and consider E an elliptic curve over \mathbb{Q} with semistable reduction at p . We show that the adjoint p -adic L -function evaluated at infinitely many integers relatively prime to p completely determines up to a quadratic twist the isogeny class of E . We also present ongoing work on the non-vanishing of the degree 4 L -function on $GSp(4)$ at the center via the relative trace formula. This constitutes one of the smallest dimension examples where the non-vanishing by abelian twists is not known at the center.

Speaker: **Jason Polák** (McGill)

Title: *Beginnings of relative endoscopy.*

Abstract: A key tool in establishing many cases of Langlands functoriality so far has been the Arthur-Selberg trace formula combined with the theory of endoscopy to express certain sums of orbital integrals on a group as sums of orbital integrals on smaller groups. Here, by group we mean the F -points of a reductive group where F is a p -adic field. For relative trace formulae used to study distinguished automorphic representations, no such theory of endoscopy exists so far. In this talk I shall explain the beginnings of such a theory.

I shall start by giving a well-motivated introduction with plenty of examples to orbital integrals, which count points on certain varieties over finite fields called affine Springer fibers. We shall discuss κ -orbital integrals and state the fundamental lemma of Ngô.

After this introduction, I shall briefly discuss the notion of relative trace formula, distinguished automorphic representations, and some results in the literature. We will discuss the problem of endoscopy in the setting of symmetric spaces. Then I will state some of my results that hint that a theory of endoscopy is indeed possible, such as a computed special case of a fundamental lemma on Lie algebras, and the calculation of the dimension of some relative affine Springer fibers.

Speaker: **Beth Romano** (Boston College)

Title: *The Local Langlands Correspondence: New Examples from the Epipelagic Zone.*

Abstract: Let G be a split reductive group over a finite extension k of \mathbb{Q}_p . The Local Langlands Correspondence (LLC) predicts that for every complex representation π of G there should be a corresponding field extension of k whose structure reflects certain properties of π . The LLC has been proven in many cases for large primes p , but remains mysterious when p is small. In recent work, Jessica Fintzen and I have found new supercuspidal representations for small p , and in some cases I have found the corresponding field extensions. In my talk I will give explicit examples for the case $G = G_2$.

Speaker: **Naser Talebizadeh Sardari** (Princeton)

Title: *Optimal strong approximation for quadratic forms.*

Abstract: For a non-degenerate integral quadratic form $F(x_1, \dots, x_d)$ in 5 (or more) variables, we prove an optimal strong approximation theorem. Fix any compact subspace $\Omega \subset \mathbb{R}^d$ of the affine quadric

$F(x_1, \dots, x_d) = 1$. Suppose that we are given a small ball B of radius $0 < r < 1$ inside Ω , and an integer m . Further assume that N is a given integer which satisfies $N \gg (r^1 m)^{4+\epsilon}$ for any $\epsilon > 0$. Finally assume that we are given an integral vector $(\lambda_1, \dots, \lambda_d) \pmod{m}$. Then we show that there exists an integral solution $x = (x_1, \dots, x_d)$ of $F(x) = N$ such that $x_i \lambda_i \pmod{m}$ and $\frac{x}{\sqrt{N}} \in B$, provided that all the local conditions are satisfied. We also show that 4 is the best possible exponent. Moreover, for a non-degenerate integral quadratic form $F(x_1, \dots, x_4)$ in 4 variables we prove the same result if $N \geq (r^1 m)^{6+\epsilon}$ and some non-singular local conditions for N are satisfied. Based on some numerical experiments on the diameter of LPS Ramanujan graphs, we conjecture that the optimal strong approximation theorem holds for any quadratic form $F(X)$ in 4 variables with the optimal exponent 4.

Speaker: **Ari Shnidman** (Boston College)

Title: *Selmer groups in families of cubic twists.*

Abstract: We give estimates for the average size of the 3-Selmer group in cubic twist families of elliptic curves. As a consequence, we conclude that for all but one of these families, the average size of 3-Selmer is infinite. We also show that 0% of curves in a given cubic twist family of genus 1 curves are everywhere locally soluble. This is joint work with Manjul Bhargava.

Speaker: **Cheng-Chiang Tsai** (MIT)

Title: *Some representation theoretic objects in number theory.*

Abstract: We describe a class of varieties which not only show up in affine Springer fibers (or, p -adic orbital integrals), but also appear in some earlier works of people on arithmetic statistics. After that we focus on some endoscopy-related identities about the (co)homology of these varieties.

ORGANIZERS:

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