Summer 2009 - Intro to Calculus
The Johns Hopkins University
Instructor: Mike Limarzi
Tuesday, July 28th, 2009

Name:

Practice Final Exam

Please show all of your work, and justify your answers completely. A correct answer with no work shown does not guarantee full points. No calculators or outside sources of assistance are permitted. You may use your brain. Don’t cheat! Good luck!
1. State the quadratic formula.

2. State the difference quotient for $f(x)$.

3. State the formula for $\sin 2x$ and $\cos 2x$.

4. State the formula for $\sin(A + B)$ and $\cos(A + B)$.

5. State the formula for $\sin(A - B)$ and $\cos(A - B)$.

6. State the half-angle formula for $\sin x$ and $\cos x$.

7. State the three Pythagorean identities.

8. State the three properties of logarithms.

9. State the change of base formula for logarithms.
10. Sketch \( y = \cos x \).

11. Sketch \( y = \sin x \).

12. Sketch \( y = \tan x \).

13. Sketch \( y = \ln x \).

14. Sketch \( y = e^x \).

15. Sketch \( y = \frac{1}{x} \).

16. Sketch \( y = \frac{1}{x^2} \).

17. Sketch \( y = x^2 \).

18. Sketch \( y = x^3 \).

19. Sketch \( y = a(x - h)^2 + k \).

20. Sketch \( (x - h)^2 + (y - k)^2 = r^2 \).
21. True or False: An odd function is one that satisfies $f(x) = -f(-x)$ for all $x$.

22. True or False: In order to be an invertible function, the graph of the curve must pass the vertical line test.

23. True or False: If the equation of a circle is $x^2 + 3x + y^2 - 4y = -\frac{9}{4}$, then the circle is centered at $(-\frac{3}{2}, 2)$ and has radius 4.

24. True or False: $\sin(A + B) = \sin A \cos B + \sin B \cos B$.

25. True or False: The change of base formula for logarithms is:

$$\log_b x = \frac{\log_a x}{\log_a b}.$$
26. Give the equation for a line that goes through the point \((1, 2)\) with slope \(\frac{1}{3}\). Give the equation of the line in both point-slope form and slope-intercept form.

27. Find the equation of a line perpendicular to \(y = -3x + 5\) through the point \((2, 1)\).

28. Find the equation of a line parallel to \(y = 3x + 2\) through the point \((2, 1)\).

29. Find all real and imaginary roots of \(x^2 + 2x + 12 = 0\). Determine when the function is positive.

30. Find all real and imaginary roots of \(x^2 + 2x - 12 = 0\). Determine when the function is positive.

31. Factor completely: \(x^7 - 3x^6 - 3x^5 + 11x^4 + 6x^3\). State the multiplicity of each root.

32. Calculate and simplify \(f(x + h) - f(x) \div h\) when \(f(x) = x^3\). After simplifying, let \(h\) tend to zero for an answer just in terms of \(x\).

33. Calculate and simplify \(f(x + h) - f(x) \div h\) when \(f(x) = x^2 + 2x + 1\). After simplifying, let \(h\) tend to zero for an answer just in terms of \(x\).
34. Is the following function invertible? If so, find its inverse. If not, explain why.
\[ f(x) = 3x + 4. \]

35. Is the following function invertible? If so, find its inverse. If not, explain why.
\[ f(x) = x^2 - 2x + 3. \]

36. Is the following function invertible? If so, find its inverse. If not, explain why.
\[ f(x) = (x - 2)^3 + 5. \]

37. Find the product: \((x - \sqrt{3}i)(x + \sqrt{3}i)\).

38. Find the product: \((2 + i)(5 - 4i)\).

39. Solve: \(\sqrt{x - 1} = x - 7.\)

40. Solve: \(\sqrt{2x + 5} + \sqrt{x + 6} = 9.\)

41. Solve: \(\left| \frac{x}{2} - 3 \right| \geq 7.\)

42. Solve: \(|4x - 5| < 2.\)
43. Simplify: $5 \ln 2 - \ln 4 + \frac{1}{3} \ln 8$.

44. Simplify: $\log_3 27^x = \log_9 81^\frac{x}{2}$.

45. Solve: $\ln(2x) - \ln(x - 15) = \ln(x - 1) - \ln(x - 7)$. Don’t forget to check answers!

46. Complete the following table with the angles that lie in the first quadrant or on the positive axes:

<table>
<thead>
<tr>
<th>$\theta^\circ$</th>
<th>$\theta$</th>
<th>$\sin \theta$</th>
<th>$\cos \theta$</th>
<th>$\tan \theta$</th>
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47. Complete the following table:

<table>
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<tr>
<th>$\theta^\circ$</th>
<th>$\theta$</th>
<th>$\cos \theta$</th>
<th>$\sin \theta$</th>
<th>$\tan \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-390^\circ$</td>
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<td>$\frac{5\pi}{11}$</td>
<td></td>
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<tr>
<td>$-45^\circ$</td>
<td></td>
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<tr>
<td>$\frac{28\pi}{3}$</td>
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</tbody>
</table>
48. Evaluate the following:
   (a.) \( \sec\left(\frac{\pi}{4}\right) \)
   (b.) \( \csc\left(-\frac{\pi}{3}\right) \)
   (c.) \( \csc^2(150^\circ) \)
   (d.) \( \cot(240^\circ) \)

49. Evaluate the following:
   (a.) \( \arcsin\left(-\frac{\sqrt{3}}{2}\right) \)
   (b.) \( \arctan(1) \)

50. Evaluate the following:
   (a.) \( \arcsin\left(\sin\left(\frac{5\pi}{4}\right)\right) \)
   (b.) \( \arctan\left(2\cos\left(\frac{5\pi}{6}\right)\right) \)

51. Accurately sketch the graph of \( y = -3\sin\left(\frac{x}{2}\right) - 1 \). Label points clearly, and give the range of the function.
52. Find all real numbers that satisfy:
\[ \sin 2x = \frac{\sqrt{3}}{2}. \]

53. Find all real numbers that satisfy:
\[ 2\cos^2 x - \cos x = 1. \]

54. Find all real numbers in the interval \([0, 2\pi]\) that satisfy:
\[ \sin^2 \left( \frac{x}{2} \right) - \frac{1}{4} = 0. \]

55. Evaluate:
\[ \sin \left( \frac{7\pi}{12} \right) \cos \left( \frac{\pi}{3} \right) - \sin \left( \frac{\pi}{3} \right) \cos \left( \frac{7\pi}{12} \right). \]

56. Is the following equation an identity? Either give a counterexample or a proof:
\[ \cos^2 x = \frac{1}{2} - \frac{\cos(2x)}{2}. \]

57. Prove that the following equation is an identity:
\[ \cos x \tan x \cot x = \cos^3 x + \cos x \sin^2 x. \]

58. Prove that the following equation is an identity:
\[ \cos^2(A - B) - \cos^2(A + B) = \sin^2(A + B) - \sin^2(A - B). \]
59. Let \( \mathbf{v}_1 = (1, 2, 3) \) and let \( \mathbf{v}_2 = (-2, 1, 5) \).

(a.) Compute \( \mathbf{v}_1 + \mathbf{v}_2 \).

(b.) Compute \( \mathbf{v}_1 \cdot \mathbf{v}_2 \).

(c.) Compute \( ||\mathbf{v}_1|| \).

(d.) Compute \( ||\mathbf{v}_2|| \).

60. Let \( A = \begin{pmatrix} 1 & 3 \\ -2 & -6 \end{pmatrix} \) and \( B = \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix} \).

(a.) Compute \( A + B \).

(b.) Compute \( A \times B \).

(c.) Compute \( B \times A \).

(d.) Are they equal?

(e.) Is \( A \) invertible? Justify.

(f.) Is \( B \) invertible? Justify.

(g.) Find \( A^{-1} \), if it exists.

(h.) Find \( B^{-1} \), if it exists.

End of practice exam.