1. (10 points) Find the extrema of \( f \) subject to the stated constraints.

\[ f(x, y, z) = x + y - 2z \quad \text{subject to} \quad x^2 + y^2 + z^2 = 24. \]

We first take the gradient of \( f = x + y - 2z \) and \( g = x^2 + y^2 + z^2 \):

\[ \nabla f(x, y, z) = (1, 1, -2) \]
\[ \nabla g(x, y, z) = (2x, 2y, 2z) \]

Now, we set them equal to each other using the Lagrange multiplier \( \lambda \):

\[ \nabla f = \lambda \nabla g \]

So, we get:

\[ 1 = \lambda 2x \quad 1 = \lambda 2y \quad -2 = \lambda 2z \]
\[ x^2 + y^2 + z^2 = 24 \]

We first notice that \( \lambda \neq 0 \), since that would imply \( 1 = 0 \) from first equation. Since \( \lambda \neq 0 \), we can set the first two equations equal to each other, then solve:

\[ \lambda 2x = 1 = \lambda 2y \quad \Rightarrow \quad x = y \]

Now, multiply the first equation by \(-2\) and setting equal to third equation, we get:

\[ -\lambda 4x = -2 = \lambda 2z \quad \Rightarrow \quad -2x = z \]

Plugging these into our fourth equation we get:

\[ x^2 + y^2 + z^2 = 24 \]
\[ x^2 + x^2 + (-2x)^2 = 24 \]
\[ x^2 + x^2 + 4x^2 = 24 \]
\[ 6x^2 = 24 \]
\[ x^2 = 4 \quad \Rightarrow \quad x = \pm 2 \]
Using our above equations we get the two points:
(2, 2, −4) and (−2, −2, 4). Plugging these points into our original $f$, we get:

$f(2, 2, −4) = 2 + 2 - 2(−4) = 12$
$f(−2, −2, 4) = −2 + −2 − 2(4) = −12$

So, our function has a max of 12 at (2, 2, −4).
Our function has a min of -12 at (−2, −2, 4).

Bonus: Name the first four Presidents of the United States, in order.
1. George Washington
2. John Adams
3. Thomas Jefferson
4. James Madison