11. (10 points) Let $F = (z \sin x, z, y - \cos x + e^{\arctan z})$.
   Let $S$ be the surface $x^2 + y^2 + z^2 = 1$.
   State and verify Stokes’ Theorem, i.e., compute both ways.

Stokes’ Theorem is:

$$\int \int_S \nabla \times F \, dS = \int_{\partial S} F \cdot ds$$

First, we shall compute $\nabla \times F$.

$$\nabla \times F = \left| \begin{array}{ccc}
   \mathbf{i} & \mathbf{j} & \mathbf{k} \\
   \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & \frac{\partial}{\partial z} \\
   z \sin x & z & y - \cos x + e^{\arctan z}
\end{array} \right| = (1 - 1) \mathbf{i} - (\sin x - \sin x) \mathbf{j} + (0 - 0) \mathbf{k} = 0$$

This means that $\nabla \times F = 0$, so,

$$\int \int_S \nabla \times F \, dS = \int \int_S 0 \, dS = 0$$

Now, we would like to verify that computing the integral the other way also gives us 0. We would like to calculate

$$\int_{\partial S} F \cdot ds$$

but, we notice that since $S$ is a closed surface, $S$ has no boundary, leaving us with:

$$\int_{\partial S} F \cdot ds = 0$$

which is what we wanted.

Bonus: Is your TA right-handed or left-handed?

I’m left-handed, but you’ve seen me write on the board all semester, so you knew that.