Weekly Quiz 12 - Thursday, December 7, 2006

1. (10 points) Let \( F = (z \sin x, z, y - \cos x + e^{\arctan z}) \).
   Let \( S \) be the surface \( x^2 + y^2 + z^2 = 1 \).
   State and verify Stokes' Theorem, i.e., compute both ways.

Stokes' Theorem is:
\[
\int \int_S \nabla \times F \, dS = \int \partial S \ F \cdot ds
\]

First, we shall compute \( \nabla \times F \).
\[
\nabla \times F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & \frac{\partial}{\partial z} \\ z \sin x & z & y - \cos x + e^{\arctan z} \end{vmatrix} = (1 - 1)\mathbf{i} - (\sin x - \sin x)\mathbf{j} + (0 - 0)\mathbf{k} = 0
\]

This means that \( \nabla \times F = 0 \), so,
\[
\int \int_S \nabla \times F \, dS = \int \int_S 0 \, dS = 0
\]

Now, we would like to verify that computing the integral the other way also gives us 0. We would like to calculate
\[
\int \partial S \ F \cdot ds
\]
but, we notice that since \( S \) is a closed surface, \( S \) has no boundary, leaving us with:
\[
\int \partial S \ F \cdot ds = 0
\]
which is what we wanted.

Bonus: Is your TA right-handed or left-handed?

I’m left-handed, but you’ve seen me write on the board all semester, so you knew that.