Calculus III - Fall 2006
Instructor - Prof. Wilson

Weekly Quiz 11 - Thursday, November 30, 2006

1. (10 points) Let $F = (y - z, z - x, x - y)$ and let $S$ be the union of $x^2 + z^2 = 4, 0 \leq y \leq 1$ and $x^2 + y^2 + z^2 = 4, y \leq 0$, oriented so that the normal at $(0, 0, 2)$ is $(0, 1, 0)$. Compute:

$$\int \int_S \nabla \times F \, dS$$

First, we see that $S$ is a cylinder along the $y$-axis with a hemisphere attached to its left end. The boundary of $S$ is the circle $x^2 + z^2 = 4, y = 1$. We can apply Stokes’ Theorem to this problem. We get:

$$\int \int_S \nabla \times F \, dS = \int_{\partial S} F \cdot ds = \int_{c} F(c(t)) \cdot c'(t) \, dt$$

where $c(t)$ is the boundary of our $S$.

Now, we must find $c(t)$. Since our boundary is a circle in two dimensions, we know how to parameterize it. Our only question is to the orientation. To determine the orientation, we use the orientation of $S$. Since $S$ has outward orientation, we can use either the right-hand rule or the man walking on the left rule to see that our circle should have a clockwise, or negative, orientation. So, we get:

$c(t) = (2 \cos t, 1, -2 \sin t), \quad 0 \leq t \leq 2\pi$
$c'(t) = (-2 \sin t, 0, -2 \cos t), \quad 0 \leq t \leq 2\pi$

Now, plugging in, we get:

$$\int_0^{2\pi} F(2 \cos t, 1, -2 \sin t) \cdot (-2 \sin t, 0, -2 \cos t) \, dt$$

$$= \int_0^{2\pi} (1 + 2 \sin t, -2 \sin t - 2 \cos t, 2 \cos t - 1) \cdot (-2 \sin t, 0, -2 \cos t) \, dt$$

$$= \int_0^{2\pi} -2 \sin t - 4 \sin^2 t + 0 - 4 \cos^2 t - 2 \cos t \, dt$$

$$= \int_0^{2\pi} -2 \sin t - 2 \cos t - 4 \, dt$$

$$= 2 \cos t - 2 \sin t - 4t \big|_0^{2\pi} = 2 - 0 - 8\pi - (2 - 0 - 0) = -8\pi$$

So, we get our answer is $-8\pi$.

Bonus: Name a type of cheese.