Section 2.5
Problem (B)

Consider a function \( f(y) \) with some finite set of zeroes. Note that \( y = y_0 \) is an equilibrium solution of \( y' = f(y) \) if and only if \( y = y_0 \) is an equilibrium solution of \( y' = -f(y) \). That is, \( y' = f(y) \) and \( y' = -f(y) \) have the same equilibrium solutions.

(i) If \( y = y_0 \) is an asymptotically stable equilibrium solution for \( y' = f(y) \), then what is its stability for \( y' = -f(y) \) ?

(ii) If \( y = y_0 \) is an unstable equilibrium solution for \( y' = f(y) \), then what is its stability for \( y' = -f(y) \) ?

(iii) If \( y = y_0 \) is a semi-stable equilibrium solution for \( y' = f(y) \), then what is its stability for \( y' = -f(y) \) ?

Problem (C)

Consider a function \( f(y) \) with a finite set of zeroes.

(i) Let \( n \) be an odd integer greater than 0. Is the stability of a solution \( y = y_0 \) to \( y' = f(y) \) always the same as the stability of \( y = y_0 \) as a solution of \( y' = f(y)^n \) ?

(ii) Let \( n \) be an even integer greater than 0. Is the stability of a solution \( y = y_0 \) to \( y' = f(y) \) always the same as the stability of \( y = y_0 \) as a solution of \( y' = f(y)^n \) ?

Problem (D)

Consider the polynomial

\[
p(y) = \prod_{k=1}^{N} (y - k)^k = (y - 1)^1(y - 2)^2(y - 3)^3...(y - N)^N
\]

and the differential equation

\[
y' = p(y)
\]

This has equilibrium solutions \( y = k \) for each \( k = 1, \ldots, N \)

(i) Which equilibrium solutions are semi-stable?

(ii) Note that the degree for \( p \) is

\[
\sum_{k=1}^{N} k = \frac{N(N + 1)}{2}
\]
For the cases:
(a) $N = 4m$
(b) $N = 4m + 1$
(c) $N = 4m + 2$
(d) $N = 4m + 3$

find the stable equilibrium solutions and the unstable equilibrium solutions. You may write your answers as patterns of numbers. In order to make sure the pattern is understood include at least 3 numbers in each pattern. (Ex: 7, 14, 21, .... ) (Hint: You will have to use the formula for the degree of $p$ to determine if the degree is odd or even. From this you know whether it is positive or negative before the root 1)