(1) Sketch a direction field for \( y' = y(5 - y)(y + 5) \)

(2) Mark the order and whether the equation is linear OR nonlinear
   (i) \( t^2y''' + \sin(t)y = e^t \cos(t) \)
   (ii) \( yy' + t = 5 \)
   (iii) \( y(4) + ty^2 = \cos t \)
   (iv) \( (t + \sin t)(y + y') = e^t \)

(3) Solve the initial value problem:
   \[ ty' - 6y = t^3 + t \]
   \[ y(1) = 1 \]

(4) Find the general solution to
   \[ y' + \cos(t)y = \cos t \]
   in terms of \( y(0) = y_0 \)

(5) Solve the following initial value problem and determine the interval of existence
   \[ y' = \frac{-4x^3}{4y^3 + 6y} \]
   \[ y(0) = -1 \]

(6) Solve the following differential equation using the substitution \( v = \frac{y}{x} \)
   \[ x^2y' = y^2 + 6xy + 6x^2 \]

(7) Using the Existence and Uniqueness Th. for Linear Equations, what is the interval of existence of the initial value problem
   \[ t(t - 3)y' + t(t - 1)y + (t - 2) = 0 \]
\( y(4) = 2 \)

(8) Consider
\[ y' = (y - 1)^2(y^2 + 1)^3(y - 3)(y - 5)^4(y - 7) \]

Draw the phase line. What are the equilibrium solutions and what are their stabilities?

(9) Consider
\[ y' = (y + a)(y^3 - a) \]

Draw the bifurcation diagram for this differential equation.

(10) Check that the following equation is exact and then solve for general solution
\[ (e^x + 3x^2y + y + y^2)dx + (x^3 + x + 2xy)dy \]

(11) Find the integrating factor \( \mu \) of
\[ (2xy)dx + (3x^2 + yx^2)dy = 0 \]
using the appropriate one of the following:
\[ \mu' = \frac{M_y - N_x}{N} \mu \]
OR
\[ \mu' = \frac{N_x - M_y}{M} \mu \]

(12) Approximate \( y(2), y(3), y(4) \) using Euler’s Method with time step \( = 1 \) for
\[ y' = 2x + y \]
(13) Using the Method of Undetermined Coefficients, solve the initial value problem
\[ y'' + 4y = e^t \]
\[ y(0) = \frac{1}{5} \]
\[ y'(0) = 1 \]

(14) Use reduction of order to find a second fundamental solution to
\[ t^2 y'' + 3ty' + y = 0 \]
\[ y_1 = \frac{1}{t} \]

(15) Consider the linear differential equation
\[ L[y] = t^2 e^t + e^{2t} \sin(t) + \cos(t) \]
with characteristic polynomial
\[ (r - 1)^5(r^2 - 4r + 5)^3(r^2 + 2)(r + 2)^2 \]

(i) What are the solutions to the homogeneous solution?
(ii) Write down guesses \( Y_1, Y_2, Y_3 \) for the Method of Undetermined Coefficients. Do NOT solve.

(16) Use variation of parameters to solve
\[ y'' + 4y = g(t) \]

(17) Consider the following differential equation:
\[ t(t - 1)^2(t + 2)^3(t + 3)y''' + (t - 1)^2 y'' + t(t + 3)y' = (t + 2)^2 \]
Using the Theorem of Existence and Uniqueness for linear equations what is the interval of existence for the following initial value problems?
(i) \( y(-1) = 1, y'(-1) = 0, y''(-1) = 0 \)
(ii) \( y(5) = 10^7, y'(5) = 0, y''(5) = 0 \)
(iii) For the same differential equation, can you say for sure without solving that the following intial value problem has a solution?
\( y(-1) = 1, y'(-1) = 1, y''(-1) = 1, y'''(-1) = 1 \)