Time Limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No Calculators.

1. There are five people at a party. Each pair of people are either friends or strangers. If two people are strangers, they do not have a mutual friend. In how many ways is this possible?

   Answer: 47

2. Daksh randomly chooses three points on a circle of radius r. What is the probability that the triangle with those three vertices contains the center of the circle?

   Answer: 1/4

3. How many nonnegative integers less than 100,000 are there with digits in nondecreasing order?

   Answer: 5005

4. You are playing a game where you continue flipping a coin until the first time you get tails. What is the probability that the game ends after a number of flips that is divisible by 3?

   Answer: 1/7

5. Jacob walks down a hallway containing a row of closed lockers numbered 1 to 2048. He opens locker 1, and alternates between skipping and opening closed lockers. When he reaches the end of the hall, he turns around and opens the first closed locker he finds and alternates between skipping and opening closed lockers. He continues going up and down the hallway in this fashion until all the lockers are opened. What is the number of the last locker he opens?

   Answer: 1366

6. Let J the number of subsets of $\{1, 2, \ldots, 2000\}$ (natural numbers up to 2000) such that the sum of the elements of the subset is divisible by 5. If $J = \frac{1}{a}2^b + \frac{1}{c}2^d$, where a, b, c, d are integers, find $(a - c + 1)(b + d)$.

   Answer: 2402

7. Two players alternate rolling a 2017-sided die. If a player's roll is less than or equal to the previous player's, they lose. The probability that the first player to roll wins the game is expressible in the form $a - (b - \frac{c}{d})^e$, where a, b, c, d, e are positive integers. Find the sum $a + b + c + d + e$.

   Answer: 4037

8. Sihyeok has 2017 buckets of water. One of the buckets is poisoned, so that if someone drinks from it, their skin turns green the following morning. Sihyeok needs to determine which bucket is poisoned by tomorrow morning. To accomplish this, he has a group of pigs that you can make to drink from the buckets. Each pig may drink from any number of buckets, and there is enough water in each bucket for any number of pigs. Sihyeok can choose the set of buckets that each pig will drink from.
   What is the smallest number of pigs he needs in order to determine which bucket is poisoned tomorrow morning, once the poison's effects occur?

   Answer: 11

9. Let

$$S = \sum_{k=0}^{2017} 2^k \binom{2017}{k} \binom{2017-k}{\lfloor (2017-k)/2 \rfloor}.$$

   If S is expressible as $\binom{a}{b}$, where a, b are positive integers, find $a + b$.

   Answer: 6052

10. On a standard mechanical watch, the location of the hour hand determines the location of the minute hand; for example, if the hour hand is halfway between two numbers on the face, the minute hand must be pointing to the 6. In a 12-hour period, how many times are there at which you could switch the location of the hour and minute hands to get another valid time reading?

    Answer: 143