

**Time Limit:** 60 minutes.

**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

**No Calculators.**

1. The probability that a person has a certain unique trait is .12. When people are tested for the trait, the test is not always accurate. If somebody has the trait, the probability that the test is correct is .96. If somebody does not have the trait, the probability that the test is correct is .92. If Jimmy tests positive for the trait, what is the probability that Jimmy has the trait? Give your answer as a simplified fraction.

Answer:  $\frac{18}{29}$

2. Given the Fibonacci sequence starting with the terms, 1, 1, 2, 3, 5 where each term is equal to the sum of the two previous terms, how many of the first 2017 terms are even?

Answer: 672

3. What is remainder of  $3^{2017}$  when divided by 7?

Answer: 3

4. There are  $x$  distinguishable permutations of the 10 letter word puzzlement. There are also  $y$  distinguishable permutations of the 10 letter word blackjacks. What is  $x/y$ ?

Answer: 2

5. Find the area of the triangle given the coordinates of three vertices of a parallelogram A  $(-32, 2)$ , B  $(3, 37)$ , and C  $(38, 2)$ .

Answer: 1,225

6. Solve the equation:  $\sqrt{x+15+8\sqrt{x-1}} + \sqrt{x+15-8\sqrt{x-1}} = 7$  (if there is no solution, write 'No solution'; if there is infinitely many solutions, write 'Infinitely many solutions').

Answer: No Solution

7. Given a parabola  $y = 2x^2$  and a line  $y = (3m+1)x - 3m + 1$ . Bob realizes that the intersection of these two lines will be to the right of  $y$ -axis as long as  $m$  is larger than a number  $k$ . Find  $k$ .

Answer:  $\frac{1}{3}$

8. How many zeros are at the end of  $1000!$

Answer: 249

9. How many bishops can one put on an  $8 \times 8$  chessboard such that no two bishops can hit each other? (Chess boards are  $8 \times 8$  grids, and bishops can move to any tile that is on a diagonal line from the tile the bishop is on).

Answer: 14

10. Find the sum of all positive integers  $x < 1000$  such that the remainder of  $x^n$  divided by 1000 is  $x$  for all positive integers  $n$ .

Answer: 1002