

**Time Limit:** 60 minutes.

**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

**No Calculators.**

1. Let  $\Gamma_1$  and  $\Gamma_2$  be circles with radii 8 and 6, respectively, such that their centers are 12 apart. Let  $P$  be one of the intersection points of  $\Gamma_1$  and  $\Gamma_2$ . A line is drawn through  $P$  such that it intersects  $\Gamma_1$  again at  $Q$  and  $\Gamma_2$  again at  $R$ , and chords  $PQ$  and  $PR$  have equal length. Find the square of the length of  $PQ$ .

Answer: 130

2. A square has sides of length 2. Set  $S$  is the set of all line segments that have length 2 and whose endpoints are on adjacent sides of the square. The midpoints of the line segments in set  $S$  enclose a region  $A$ . Find the area of  $A$ .

Answer:  $4 - \pi$

3. A rectangle inscribed in a larger rectangle is called “free” if it is possible to rotate (even slightly) the smaller rectangle about its center within the boundaries of the larger rectangle. Of all rectangles that can be inscribed free in a  $6 \times 8$  rectangle, let the smallest perimeter be  $P$ . Find  $P^2$ .

Answer: 448

4. If the sides of a triangle are  $2x + 3$ ,  $x^2 + 3x + 3$ , and  $x^2 + 2x$ , and  $x > 0$ , find the greatest interior angle of a triangle (in degrees).

Answer: 120

5. Find the area of a square ABCD containing a point P such that  $PA = 3$ ,  $PB = 7$ , and  $PD = 5$ .

Answer: 58

6. In  $\triangle ABC$ ,  $BB'$  and  $CC'$  are medians.  $D$  is on  $BB'$  such that the intersection point of  $AD$  and  $BB'$ ,  $M$ , is the midpoint of  $BB'$ .  $N$  is the intersection of  $AD$  and  $CC'$ . What is  $\frac{|C'N|}{|NC|}$ ?

Answer:  $1/4$

7. In  $\triangle ABC$ , the medians  $AD$  and  $CE$  have lengths 18 and 27, respectively, and  $AB = 24$ . Extend  $CE$  to intersect the circumcircle of  $ABC$  at  $F$ . Find the the square of the area of  $\triangle AFB$ .

Answer: 3520

8. Two circles,  $\Gamma_1, \Gamma_2$  are tangent internally at  $P$ , and a chord,  $AB$ , of the larger circle  $\Gamma_1$  is tangent to the smaller circle  $\Gamma_2$  at  $C$ . Chords  $PB$  and  $PA$  intersect  $\Gamma_2$  again at  $E$  and  $D$ , respectively. If  $|AB| = 15$ , while  $|PE| = 2$  and  $|PD| = 3$ , find  $|AC|$ .

Answer: 9

9. Let  $\triangle ABC$  be a nonobtuse triangle such that  $AB > AC$  and  $\angle B = 45^\circ$ . Let  $O$  be the circumcenter of  $ABC$  and let  $I$  be the incenter of  $ABC$ . Suppose that  $\sqrt{2}|OI| = |AB| - |AC|$ . Determine the sum of the values of  $\sin^2 A$ .

Answer:  $\sqrt{2}$

10. Let  $\triangle ABC$  be nonobtuse. Let  $P_4$  be a square,  $P_m$  be a  $m$ -sided regular polygon, and let  $P_n$  be a  $n$ -sided regular polygon, where  $P_4, P_m, P_n$  each share a side with the triangle  $ABC$ , and none of the four shapes overlap. The centers of  $P_4, P_m, P_n$  form an equilateral triangle. Find  $m + n$ .

Answer: 16