

**Time Limit:** 60 minutes.

**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

**No Calculators.**

1. What is  $2017^{2016^{2015^{\dots^1}}}$  mod 10?

Answer: 1

2. For which positive integer  $c$  will  $(c^6 - 3)/(c^2 + 2)$  also be an integer?

Answer: 3

3. How many zeros are at the end of  $1000!$

Answer: 249

4. Find the sum of all primes that are of the form  $n^3 - 1$  for some integer  $n$ ?

Answer: 7

5. How many bishops can one put on an  $8 \times 8$  chessboard such that no two bishops can hit each other?

Answer: 14

6. All the integers beginning with 1 are written successively (that is, 1234567891011121314...). What digit occupies the 206,788th position?

Answer: 7

7. Label one disc '1', two discs '2', three discs '3', ..., fifty discs '50.' Put these  $1 + 2 + 3 + \dots + 50 = 1275$  labeled discs in a box. Discs are then drawn from the box at random without replacement. What is the minimum number of discs that must be drawn in order to guarantee drawing at least ten discs with the same label?

Answer: 415

8. What is  $10^{10} + 10^{10^2} + \dots + 10^{10^{10}}$  mod 7?

Answer: 5

9. Find the sum of all positive integers  $x < 1000$  such that the remainder of  $x^n$  divided by 1000 is  $x$  for all positive integers  $n$ .

Answer: The numbers are 1, 376, and 625 so the total is 1002.

10. Find the sum of all primes among the positive integers, such that when they are written as usual in base 10, are alternating 1s and 0s, beginning and ending with 1?

Answer: 101