

Time Limit: 60 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No Calculators.

1. The circle $x^2 + y^2 = 20$ intersects the parabola $y = x^2$ at two points, (a, b) and (c, d) . Compute $ac + bd$.
Answer: 12
2. x is an odd number that is the sum of two prime numbers less than 100. y is an even number that is the sum of two prime numbers less than 100. If $xy = 100$, are the possible values of x/y ?
Answer: $\frac{1}{4}$ and $\frac{25}{4}$
3. Harry, Ron, Hermione, Neville, Draco, Albus, Severus, and Muggle Steve are to pose in a row for a photograph. If Harry and Steve refuse to stand next to each other, how many arrangements are possible?
Answer: 30240
4. Suppose six slips of paper, labeled 1 to 6, are placed in a hat. Three slips of paper are selected without replacement from the hat. The first slip of paper selected is the radius of a circle, and the second and third slips are the x and y coordinates of the center of the circle, respectively. Find the probability that part of the circle lies in the third quadrant.
Answer: $\frac{3}{10}$
5. Find a 4-digit perfect square so that if we increases every digit of this number by 1, the new number will also be a perfect square.
Answer: 2025
6. Given line equation $2x - y = 20$. When slope of line is increased by 150% and y -intercept increased by 10, this new line passes through points $(x, 10)$ and $(10, y)$. What is value xy ?
Answer: 160
7. Label one disc '1', two discs '2', three discs '3', ..., fifty discs '50.' Put these $1 + 2 + 3 + \dots + 50 = 1275$ labeled discs in a box. Discs are then drawn from the box at random without replacement. What is the minimum number of discs that must be drawn in order to guarantee drawing at least ten discs with the same label?
Answer: 415
8. Given $x > 0$, $y > 0$, and $x + y = 1$. Find the minimum value of the equation: $((x+1)/y)^2 + ((y+1)/x)^2$.
Answer: 18
9. Given that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ find the value of $\sum_{m=1,3,5,\dots}^{\infty} \frac{1}{m^2}$, where m ranges over all odd positive integers. Answer: $\frac{\pi^2}{8}$
10. Let $f(x) = |x - p| + |x - 17| + |x - p - 17|$, where $p \leq x \leq 17$. Find the minimum value of $f(x)$ in the interval $0 < x \leq 17$.
Answer: 17