

**Time Limit:** 60 minutes.

**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

**No Calculators.**

Useful Formula:  $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

1. Calculate:  $\lim_{x \rightarrow \infty} \sqrt{x^2 + x} - x$   
 Answer:  $\frac{1}{2}$

2. Compute the following definite integral:

$$\int_{-\sqrt[3]{\pi}}^{\sqrt[3]{\pi}} x^2 \sin(\sin(x^3)) dx$$

Answer: 0

3. Compute the following sum:

$$\sum_{n=0}^{\infty} \frac{1}{\int_0^{\infty} x^n e^{-x} dx}$$

Answer:  $e$

4. Ash has a piece of paper in the shape of rectangle  $ABCD$ , with  $AB > BC$ . Ash marks a point  $P$  on edge  $CD$  so that  $CP \leq BC$ . Ash folds the paper so that vertex  $B$  coincides with point  $P$ . Let the point on  $BC$  that is closest to  $B$  but not a part of the fold be point  $E$ . Find the ratio of  $CP$  to  $BC$  that will maximize the area of the triangle  $CEP$

Answer:  $\frac{\sqrt{3}}{3}$

5. Misty has a paper circle of radius  $r$ , that she wishes to turn into a cone. Misty accomplishes this by cutting the circle at two radii separated by angle  $\theta < 2\pi$  and folding up the remaining paper so that the places where she cut are touching one another. Find  $\theta$  so that Misty's cone has the largest possible volume.

Answer:  $2\pi \left(1 - \sqrt{\frac{2}{3}}\right)$

6. Two hallways run perpendicular to one another and meet at a corner. One hallway has width  $27ft$  and the other has width  $64ft$ . If Brock wants to bring a ladder around the corner, and assuming the ladder must be carried parallel to the floor and has no width, what is the longest ladder he could bring?

Answer: 125

7. Consider the following definite integral:

$$\int_9^{64} \frac{\sqrt{1 + \sqrt{x}}}{x} dx$$

The value of this integral can be expressed in the form  $a + \ln\left(\frac{p}{q}\right)$  where  $p$  and  $q$  are in relatively prime. Find  $a + p + q$ .

Answer: 15

8. Let  $k$  be an integer greater than 1. Compute the following integral:

$$\int_0^{\infty} \frac{dx}{(x^2 + 1)(x^k + 1)}$$

Answer:  $\frac{\pi}{4}$

9. Compute the following definite integral

$$\int_0^{\pi} \frac{1}{1 + \sin(x)} dx$$

Answer: 2

10. Compute the following definite integral:

$$-6 \int_0^1 \frac{\ln(1-x)}{x} dx$$

Answer:  $\pi^2$