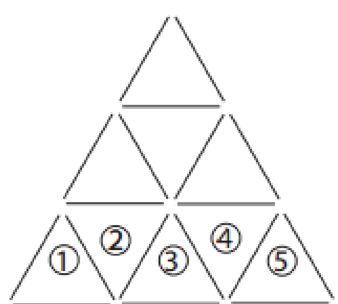
1. Find a positive integer N such that $N \leq 1000$ and N has at least 31 divisors, including N and 1.
   Answer: 840

2. Dexter and Dee Dee are playing a game. Each of them picks a number between 1 and 5, inclusive. If they pick the same number, the game ends; if not, they throw out the numbers they picked and try again. Dexter wins if and only if the game ends the second time they pick. What is the probability that Dexter wins?
   Answer: 4/25

3. You are in an underground cavern with three possible exits, which are called paths A, B, and C. Path A will lead you back to where began after 4 hours of travel; path B will lead you back to where you began after 6 hours; path C will lead you to the surface in 8 hours. However, you don't know which path is which, and every time you return to where you began, you forget which path you picked last time. What is the expected time in hours it will take before you reach the surface?
   Answer: 18

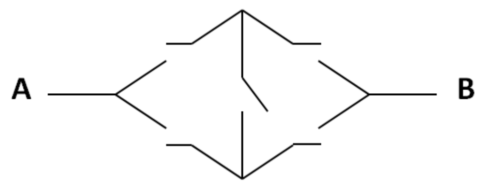
4. On an 8x8 chessboard, two squares are chosen randomly. What is the probability that they have a side in common?
   Answer: 1/36

5. A large equilateral triangle is constructed by using toothpicks to create rows of small equilateral triangles. For example, the figure shows 3 rows of small equilateral triangles, with 5 small triangles in the base row. How many toothpicks would be needed to construct such a large equilateral triangle if the base row of the triangle consists of 2017 small equilateral triangles?



   Answer: 3,054,243=$3(1009)^2$

6. In the figure, the five switches operate independently, and are each closed with probability $\frac{1}{2}$. What is the probability that there is a path from A to B?



Answer: 1/2

7. Four brother/sister pairs are seated around a round table randomly. Given that the seating alternates between men and women, what is the probability that no brother/sister pair sits next to each other?
Answer: 1/12

8. 100 people line up to board an airplane. Each has a boarding pass with an assigned seat. However, the first person to board has lost his boarding pass and takes a random seat. After that, each person takes their assigned seat if it is unoccupied, and one of the unoccupied seats at random otherwise. What is the probability that the last person to board gets to sit in their assigned seat?
Answer: 1/2

9. Determine the number of lists $(x_1, \ldots x_n)$ such that each x_i is a non-negative integer and

$$\sum_{i=1}^{n} x_i \leq k$$

where $k \geq n$. Leave your answer in terms of k and n. Lists containing the same numbers in a different order should be counted separately.
Answer: $\binom{k+n-1}{k}$

10. Ten points are placed on a circle such that no three chords with distinct endpoints chosen from the ten points intersect at a single point. The ten points are randomly labeled $A, B, C, ..., J$. What is the probability that the chords $AB, CD, EF, GH, andIJ$ divide the circle into 15 distinct regions?
Answer: $\frac{1}{189}$