

Time limit: 60 minutes.

No calculators.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Your answers may contain trigonometric expressions. Only answers written inside the boxes on the answer sheet will be considered for grading.

1. A tessellation is created by filling a plane using geometric shapes with no overlaps and no gaps. How many unique tessellations can be created that use only 1 regular polygon?
Answer: 3
2. Triangle XYZ is pictured to the right with $YZ = 5$, $YB = 3$, and $XA = 4$. What is the length of XY ?
Answer: $\frac{\sqrt{145}}{3}$
3. The length of the longer base of trapezoid $ABCD$ is 97. The length of line segment \overline{XY} , which joins the midpoints of diagonals \overline{AC} and \overline{BD} , is 3. Find the length of the shorter base of $ABCD$.
Answer: 91
4. The diameter \overline{CD} of a circle is extended through D to a point P outside the circle. The measure of secant \overline{CP} is 77. From P , another secant is drawn which intersects the circle at points A and B . The length of secant \overline{PB} is 33 and the diameter \overline{CD} is 74. What is the degree of $\angle APD$?
Answer: 60
5. Circle A has radius 6. 12 identical squares with side length 4 are placed such that their centers are equally spaced out on Circle A . Each square is oriented in the same way and exactly 4 of the squares are oriented such that a line drawn from the center of the circle to the center of the square would be perpendicular to the side of the square that it crosses. What is the total area covered by the squares if intersections are only counted 1 time?
Answer: $144\sqrt{3} - 96$
6. Point Z is the midpoint of \overline{AC} and \overline{AY} is a median of $\triangle ABC$. Point X is on \overline{AB} such that $AX = \frac{AB}{4}$. \overline{XZ} and \overline{AY} meet at Point M . Find $\frac{AM}{AY}$.
Answer: $\frac{1}{3}$
7. Circle A has radius 5 and is centered at the origin. Circle B has radius 4 and is centered at the point $(9, 0)$. If circle C has radius 3 and is centered in the first quadrant, at what point should it be centered in order for it to be tangent to both Circle A and Circle B ?
Answer: $\left(\frac{16}{3}, \frac{8\sqrt{5}}{3}\right)$
8. Circle A has radius 2 and is centered at $(-1, 0)$. Circle B has radius 2 and is centered at $(1, 0)$. What is the area of their intersection?
Answer: $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$
9. Isosceles triangle $\triangle ABC$ has base \overline{BC} of length 4 and sides \overline{AB} and \overline{AC} of length 3. A line is drawn through points X and Y , which lie on the base and one side (not on the vertices), respectively. \overline{XY} divides both the perimeter and the area in half. Find the length of \overline{XY} .
Answer: $\sqrt{5}$
10. Regular pentagon $ABCDE$ is pictured to the right. A 5-pointed star is drawn with each point at a midpoint of a side of the pentagon, designated by points F, G, H, I , and J . The center of this star is regular pentagon $KLMNO$. What fraction of the area of pentagon $ABCDE$ is occupied by pentagon $KLMNO$?
Answer: $\cos^2 72 \approx .0955$