

**Time limit:** 60 minutes.

**No calculators.**

**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Your answers may contain trigonometric expressions. Only answers written inside the boxes on the answer sheet will be considered for grading.

1. The Powerpuff Girls are shooting laser beams at Mojo Jojo. If the probability that they hit him with the  $n$ th laser is  $\frac{n}{n+2}$ , what is the probability that they hit him with every one of their first 13 laser beams?
2. Raph, Leo, Donnie, and Mickey are making a 1 square foot pizza together. After rolling out the dough, they throw exactly 37 pieces of pepperoni randomly at the pizza. What is the likelihood that the centers of at least two pepperonis are within 3 inches of each other?

3. Suppose

$$28x + 30y + 31z = 365.$$

What is  $z - 2x$ ?

4. A magic square is a square array of numbers such that the sum of the numbers in each row, each column, and the two diagonals is the same. The nine numbers 7, 10, 13, ..., 31 can be used to complete a three by three magic square. What is the sum of the entries of a row of such a magic square?
5. Let  $M$  and  $N$  denote the two integers that are respectively twice and three times the sum of their digits. What is  $M + N$ ?
6. Find the sum of the given series if  $m$  is a positive integer.

$$\sum_{n=1}^m \cos \frac{n\pi}{m}.$$

7. Sailor Moon is rolling a die. If she gets a five, she stops rolling. If she gets anything else, she rolls again. What is the probability that she stops rolling after an even number of rolls?
8. Find the remainder of  $2^{50}$  when divided by 17.
9. The Smurf population in Smurf Village at one time was a perfect square. Later, after an increase of 100 Smurfs, the population was one more than a perfect square. Now, another 100 Smurfs have joined so the population is once again a perfect square. What was the original population of Smurf Village?
10. Suppose Doug Funnie has 12 coins and knows that 11 of them are of the same weight and 1 of them is of a different weight. He has a two-pan balance and wants to figure out which coin differs in weight and whether it is heavier or lighter than the other 11 coins. What is the minimum number of weighings required for Doug Funnie?