

Time limit: 60 minutes.

No calculators.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

1. The line $y = -2x + 3$ intersects the parabola $y = x^2$ at point A and point B . What is the area of the triangle formed by joining A , B , and the origin?
2. Let k be a positive integer no greater than 1000. k is divisible by the smallest odd prime number, but k is not divisible by the largest one-digit prime number or the smallest two-digit prime number. How many such k exist?
3. A hypothetical genetic disease is inherited in this way: (1) It can only be inherited from the mother. (2) If the parent is afflicted with the disease, a child has a $1/2$ chance to be a carrier (does not show disease symptoms) and a $1/4$ chance to be afflicted. (3) If a parent is a carrier, the child has a $1/3$ chance to be a carrier and a $1/6$ chance to be afflicted. Angelica (female) and Chucky (male) are a young couple and about to have a child. Angelica's maternal grandmother and Chucky's mother are both afflicted with this disease, however Angelica, Chucky, and Angelica's mother have never been tested for the disease and thus do not know if they are afflicted, carriers, or healthy. All of the other people in their families are neither afflicted nor are they carriers. What is the chance that Angelica and Chucky's child will be afflicted with the disease?
4. He-Man is walking north on the street at constant speed. He notices that every six minutes he is passed by a bus going north, and every three minutes he is passed by a bus going south. Assume that all buses follow the same route, leave from the same station, and move at the same speed. How long after one bus leaves the bus station does the next one leave in minutes?
5. Skeletor has 22 candies with three flavors: 6 are strawberry, 7 are orange, and 9 are mint. She now grabs three candies at random. What is the probability that the three candies are of exactly two different flavors?
6. Given that a is a positive integer, the following quadratic equation has at least one root.

$$ax^2 + 2(2a - 1)x + 4(a - 3) = 0.$$

How many unique values of a exist?

7. Let f be a function such that $f(1) = 5$ and $f(xy + 1) = \frac{1}{2}f(x)f(y) - 3x - 3y + \frac{1}{2}$ for all real values of x and y . Find $f(10)$.
8. In rectangle $ABCD$, $AB = 10$ and $BC = 8$. E is a point on AD and F is a point on CD . AF intersects BE at point O . Suppose $AF \perp BE$ and $DF = 6$, find OE .
9. A number of students have participated in a chess tournament. The tournament was held in a single round-robin style; each student played every other student once. In every game, the winner received three points and the loser received no points. In the case of a tie, both students received one point. At the end of the tournament, the total score of all students was 121 and fewer than 50% of the games were ties. Find the number of participants.
10. Find the real number x that satisfies the given equation.

$$\sqrt{2x^2 - 1} + \sqrt{x^2 - 3x - 2} = \sqrt{2x^2 + 2x + 3} + \sqrt{x^2 - x + 2}.$$