Time Limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No Calculators.

1. Let (m, n) the positive integer solution to $20m + 10 + 6n = 2016$ such that $|n - m|$ is minimized. Find $m + n$.

2. Gohan has a very large number of lollipops in an appropriately large bag and wants to share it among his friends. Initially, he splits his candy evenly with 15 friends and himself and find there are 7 lollipops left over. Then, Krillin, Dende, and Roshi arrive and they redistribute the lollipops, but there are 3 left over. If the bag has more than 500 lollipops, what is the smallest number of lollipops it can have?

3. How many real solutions does the following equation have?
$$2014^x + 2015^x = 2016^x$$

4. Let x, y, z be real numbers such that
$$\frac{x-y}{z} + \frac{y-z}{x} + \frac{z-x}{y} = 64.$$
Find
$$2016 - \frac{x-y}{z} \cdot \frac{y-z}{x} \cdot \frac{z-x}{y}.$$

5. Suppose f satisfies $f(x) + f(x-1) = x^2$ and $f(17) = 85$. Find $f(85)$.

6. Find the sum of the real solutions of $x^6 - 14x^4 - 40x^3 - 14x^2 + 1 = 0$.

7. Consider sequences of 2016 positive numbers such that their sum and the sum of their reciprocals is 2017. Let $x = \frac{m-\sqrt{n}}{q}$ be the number in **any** of these sequences that maximizes $x + \frac{1}{x}$. Find mnq.

8. Let $f(x)$ be defined for positive reals such that $f(x) = x \cdot \lfloor x \cdot \lfloor x \rfloor \rfloor$. If $y = \frac{m}{n}$ is the solution, in lowest terms, to $f(y) = 1875$, find $m - n$.

9. Let an integer a satisfy $\sqrt{a + \sqrt{a + \sqrt{a + \cdots}}} - \cfrac{1}{a + \cfrac{1}{a + \cfrac{1}{a + \ddots}}} > 10$. Find the smallest integer value of a.

10. If $0 \leq \theta \leq \pi$ and $\sin(\frac{\theta}{2}) = \sqrt{1 + \sin\theta} - \sqrt{1 - \sin\theta}$, the sum of the possible values of $\tan(\theta)$ (in lowest terms) is of the form $-\dfrac{a}{b}$ where a and b are positive integers. Find $a + b$.